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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

SIMPLIFIED METHOD OF DETERMINING TRUE BEARINGS OF A LINE

By Philip L. Inch,[1] Assoc. M. Am. Soc. C. E.

Synopsis

A simplified method of computing a direct observation of the sun, for azimuth, is offered in this paper. It involves reference to a comprehensive table specially prepared for this purpose. The objects of the table are:

(1) To place the determination of a meridian within the attainment of engineers, unprepared, by reason of the pressure of their usual work;

(2) To obviate textbook reference, at the time a meridian is necessary;

(3) To displace the inaccuracy of compass meridians in field conditions where the compass is affected by local attraction;

(4) To reduce the time necessaary to compute a direct solar observation; and

(5) To reduce trigonometric formulas to simple arithmetic.

Theory

Problems involving the determinations of true bearings may be solved by applying the formula,[2]

$$\cos Z = \frac{\sin \delta}{\cos h \cos \theta} - \tan h \tan \phi \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which Z = the azimuth of the sun; δ = the sun's declination; h = its altitude; and ϕ = the latitude of the point of observation. Let $\frac{\sin \delta}{\cos h \cos \theta} = A$; and, $\tan h \tan \phi = B$; then, Equation (1) becomes, simply,

$$\cos Z = A - B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Note—Discussion on this paper will be closed in **December, 1936,** *Proceedings*.

[1] U. S. Surveyor, Helena, Mont.

[2] "Surveying Theory and Practise", by R. E. Davis, F. S. Foote, and W. H. Rayner, Members, Am. Soc. C. E., McGraw-Hill Book Co., 1928, p. 425.

TABLE 1.—Values of Factors A and B, for a Given Altitude of the Sun, at a Given Latitude on the Earth

Vertical angle, h	Symbol	Latitude, $\lambda = 37°$: Factor	Differences for 1′: h	Differences for 1′: λ	Latitude, $\lambda = 38°$: Factor	Differences for 1′: h	Differences for 1′: λ	Latitude, $\lambda = 39°$: Factor	Differences for 1′: h	Differences for 1′: λ	Latitude, $\lambda = 40°$: Factor	Differences for 1′: h	Differences for 1′: λ	Latitude, $\lambda = 41°$: Factor	Differences for 1′: h	Differences for 1′: λ	Latitude, $\lambda = 42°$: Factor	Differences for 1′: h	Differences for 1′: λ
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
21°.......	A	1.34085	15.5	30.1	1.35893	15.7	31.7	1.37793	15.9	33.3	1.39790	16.1	35.0	1.41889	16.3	36.8	1.44097	16.6	38.7
	B	0.28866	25.3	17.7	0.29929	26.3	18.2	0.31020	27.3	18.7	0.32143	28.3	19.3	0.33300	29.3	19.9	0.34492	30.3	20.5
22°.......	A	1.35012	16.3	30.3	1.36832	16.6	31.9	1.38745	16.8	33.5	1.40756	17.0	35.2	1.42869	17.3	37.1	1.45093	17.6	39.0
	B	0.30388	25.7	18.7	0.31507	26.7	19.2	0.32656	27.6	19.7	0.33838	28.7	20.3	0.35056	29.7	20.9	0.36310	30.7	21.6
23°.......	A	1.35992	17.3	30.6	1.37826	17.5	32.1	1.39752	17.8	33.8	1.41778	18.0	35.5	1.43907	18.3	37.3	1.46146	18.6	39.3
	B	0.31932	26.1	19.6	0.33106	27.2	20.2	0.34314	28.0	20.7	0.35557	29.1	21.3	0.36836	30.1	22.0	0.38154	31.2	22.7
24°.......	A	1.37028	18.3	30.8	1.38875	18.6	32.4	1.40817	18.8	34.0	1.42858	19.1	35.7	1.45002	19.4	37.6	1.47259	19.7	39.6
	B	0.33497	26.5	20.6	0.34730	27.5	21.1	0.35996	28.5	21.7	0.37300	29.4	22.4	0.38642	30.6	23.1	0.40025	31.7	23.8
25°.......	A	1.38127	19.2	31.0	1.39989	19.4	32.6	1.41946	19.7	34.4	1.44004	20.0	36.0	1.46166	20.3	37.9	1.48440	20.6	39.9
	B	0.35087	26.9	21.5	0.36378	27.9	22.1	0.37705	28.9	22.8	0.39070	30.0	23.4	0.40476	31.1	24.1	0.41924	32.2	24.9
26°.......	A	1.39277	20.3	31.3	1.41154	20.6	32.9	1.43128	20.9	34.6	1.45202	21.2	36.3	1.47382	21.5	38.2	1.49676	21.8	40.2
	B	0.36702	27.4	22.5	0.38054	28.4	23.1	0.39441	29.5	23.8	0.40869	30.5	24.5	0.42340	31.6	25.3	0.43855	32.8	26.1
27°.......	A	1.40495	21.4	31.6	1.42389	21.7	33.2	1.44380	22.0	34.9	1.46472	22.3	36.7	1.48671	22.6	38.6	1.50985	23.0	40.6
	B	0.38347	27.9	23.5	0.39758	28.9	24.2	0.41208	30.0	24.9	0.42700	31.1	25.6	0.44236	32.2	26.4	0.45820	33.3	27.2
28°.......	A	1.41777	22.5	31.9	1.43689	22.6	33.5	1.45697	23.1	35.2	1.47809	23.4	37.0	1.50028	23.8	38.9	1.52363	24.2	41.0
	B	0.40020	28.4	24.6	0.41493	29.5	25.2	0.43006	30.5	26.0	0.44563	31.6	26.7	0.46166	32.8	27.5	0.47818	34.0	28.4
29°.......	A	1.43126	23.7	32.2	1.45055	24.0	33.8	1.47083	24.4	35.5	1.49215	24.7	37.3	1.51455	25.1	39.3	1.53812	25.5	41.4
	B	0.41724	29.0	25.6	0.43260	30.0	26.3	0.44838	31.1	27.1	0.46461	32.2	27.9	0.48133	33.4	28.7	0.49855	34.6	29.7
30°.......	A	1.44547	24.9	32.5	1.46496	25.2	34.1	1.48544	25.4	35.9	1.50697	26.0	37.7	1.52959	26.4	39.7	1.55340	26.8	41.8
	B	0.43461	29.5	26.7	0.45061	30.6	27.4	0.46704	31.8	28.1	0.48395	32.9	29.0	0.50136	34.1	29.9	0.51931	35.3	30.9
31°.......	A	1.46041	26.2	32.8	1.48010	26.5	34.5	1.50079	26.9	36.3	1.52254	27.3	38.1	1.54540	27.7	40.1	1.56945	28.2	42.2
	B	0.45233	30.1	27.8	0.46898	31.3	28.5	0.48609	32.4	29.3	0.50369	33.6	30.2	0.52181	34.8	31.1	0.54048	36.1	32.1
32°.......	A	1.47612	27.5	33.2	1.49602	27.5	34.9	1.51694	28.2	36.6	1.53892	28.7	38.5	1.56203	29.1	40.5	1.58634	29.5	42.1
	B	0.47043	30.8	28.9	0.48775	32.0	29.7	0.50554	33.1	30.5	0.52384	34.3	31.4	0.54269	35.6	32.4	0.56211	36.9	33.4
33°.......	A	1.49261	28.9	33.6	1.51274	29.3	35.2	1.53388	29.7	37.1	1.55611	30.2	39.0	1.57948	30.6	41.0	1.60406	31.1	43.1
	B	0.48893	31.2	30.0	0.50693	32.7	30.8	0.52542	33.9	31.7	0.54444	35.1	32.7	0.56403	36.4	33.7	0.58422	37.7	34.7
34°.......	A	1.50997	30.2	33.9	1.53032	30.7	35.7	1.55172	31.1	37.5	1.57421	31.5	39.4	1.59784	32.0	41.5	1.62271	32.5	43.6
	B	0.50786	32.3	31.2	0.52656	33.5	32.0	0.54576	34.7	32.9	0.56552	35.9	33.9	0.58587	37.2	34.9	0.60683	38.6	36.1
35°.......	A	1.52811	32.0	34.3	1.54871	32.5	36.1	1.57036	32.9	37.9	1.59312	33.4	39.9	1.61704	33.9	42.0	1.64221	34.4	44.1
	B	0.52723	33.1	32.4	0.54664	34.3	33.2	0.56657	35.6	34.2	0.58708	36.9	35.2	0.60821	38.2	36.3	0.62997	39.5	37.5

The writer has developed a complete table, of which Table 1 is a very condensed form, intended only for the solution of the illustrative example in the paper. By full discussion it is hoped to establish the most desirable form for the complete tables, with the possibility (only tentative) of including such revised tables, in their most convenient form, as an Appendix to the closing discussion.

Field Procedure

The field party will need a transit with a full vertical circle, an ephemeris of the sun (obtainable from the Superintendent of Public Documents, United States Printing Office, or from most dealers in engineering instruments), a table of sines and cosines, and the latitude of the point of observation, as scaled from an accurate map.

Example 1.—To find the true bearing of Line XY (Fig. 1): (1) Set the transit firmly at Point X,; (2) set the horizontal graduated plates at 0°; (3) level the instrument with extreme care; (4) sight on Point Y; (5) clamp the lower plate; and (6) loosen the upper plate. Using a solar prism or colored glass as a sunshade on the telescope, place the vertical and center horizontal cross-hairs, tangent, respectively, to the right and lower limbs of the sun as shown in Fig. 2(*a*). The observations are recorded as shown in Table 2, Item No. 1. Reverse the instrument, place the vertical and center horizontal cross-hairs tangent to the sun's left and lower limbs, respectively, and, as before, record the time, the vertical angle, and the horizontal angle, as shown in Table 2, Item No. 2. The mean of the recorded quantities (Item No. 3, Table 2) gives the time when the center of the of the sun has a certain elevation above the earth's horizon and is at a certain horizontal angular distance from Line XY.

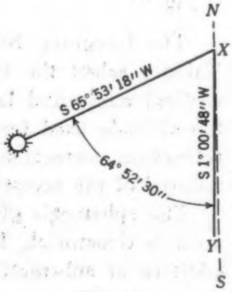

FIG. 1.

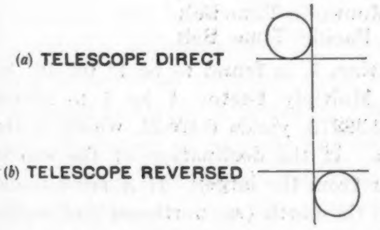

FIG. 2.

TABLE 2.—Record of Observations

(Place: Washington, D. C.; Latitude: 38° 53′ 40″ N; Eastern Time Belt; Date: March 18, 1910 (P. M.); and Point Y (Fig. 1) to the Left of the Sun.)

Item No.	Telescope position (see Fig. 2)	Time	Vertical angle	Horizontal angle
1.....	Direct	3 hr 53 min 05 sec	25° 20′ 00″	65° 00′ 00″
2.....	Reversed	3 hr 54 min 55 sec	25° 31′ 00″	64° 45′ 00″
3.....	Mean	3 hr 54 min 00 sec	25° 25′ 30″	64° 52′ 30″

Concisely, the computations may be arranged as follows:

Substitution Factors =		*A*	*B*
$h = 25°; \lambda = 38°$ (see Table 1)		1.39989	0.36378
$+h = 0°\ 25'\ 30''$		$495 = 25\frac{1}{2} \times 19.4$	$712 = 25\frac{1}{2} \times 27.9$
$+\lambda = 0°\ 53'\ 40''$		$1750 = 53\frac{2}{3} \times 32.6$	$1186 = 53\frac{2}{3} \times 22.1$
Total		1.42234	0.38276
$\sin \delta$	=	$\times$ 0.01811	......
$A \sin \delta$	=	0.025759	......
Natural cosine, bearing of the sun..................	=	0.38276 − 0.02576 = 0.40852	
Bearing of the sun.........	=	S 65° 53′ 18″ W	
Horizontal angle (see Table 2)	=	64° 52′ 30″	
Bearing of Line *XY* (see Fig. 2)..................	=	S 1° 00′ 48″ W	

The foregoing computations may be further explained as follows: From Table 1 select the values of *A* and *B* corresponding to the even degree of vertical angle and latitude, refining both for the additional minutes. Since the altitude used for computing Table 1 is obtained by applying the average refraction correction for the corresponding vertical angle, the computer is relieved of the necessity of applying the refraction correction and the parallax.

The ephemeris gives the declination of the sun, either north or south, at noon in Greenwich, England, and also gives an hourly change, which is either additive or subtractive. This declination at noon is the declination of the sun at:

7:00 A. M. within the Eastern Time Belt
6:00 A. M. within the Central Time Belt
5:00 A. M. within the Mountain Time Belt
4:00 A. M. within the Pacific Time Belt

Thus, in Example 1, the sun's declination, δ, is found to be 1° 02′ 16″ S; and the natural sine of $\delta = 0.01811$. Multiply Factor *A* by δ to obtain 0.02576, and this value, added to $B = 0.38276$, yields 0.40852, which is the natural cosine of the bearing of the sun. If the declination of the sun is north, subtract $A \sin \delta$ and *B*, the smaller from the larger. If $A \sin \delta$ is the larger, the bearing of the sun will be from the north (as, northeast and northwest); and, if *B* is the larger, the bearing of the sun is from the south (as, southeast and southwest). If the declination, δ, is south, add $A \sin \delta$ and *B*, and the bearing of the sun is from the south (as, southeast and southwest).

Finally, to the bearing of the sun, apply the horizontal angle to obtain the bearing of Line *XY*, S 1° 00′ 48″ W, allowing a probable error of 1′.

Conclusion

The writer disclaims all intention of introducing a "new method" to the Engineering Profession; his intention is simply to reduce to the commonest wording the intellectual niceties of an astronomical observation. The method of observation described herein is an old one; yet arranged more nearly in keeping with the elementary design of the paper. Any suggestions for the improvement of Table 1, compatible with its intent and purpose, will be appreciated.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

ANALYSIS OF CONTINUOUS FRAMES BY BALANCING ANGLE CHANGES

By L. E. Grinter[1], Assoc. M. Am. Soc. C. E.

Synopsis

The interest that has been shown by the profession in the method of balancing fixed-end moments (introduced by Hardy Cross, M. Am. Soc. C. E.[2]) as an alternative to the classic methods of analyzing continuous frames, has suggested the publication of a second alternative process based upon the conception of balancing angle changes to obtain joint rotations. The procedure of balancing angle changes is shown to parallel the procedure used in balancing and distributing fixed-end moments. No claim is made that the suggested method surpasses the method of moment distribution either in simplicity or in a reduction of time consumed by the analysis. In fact, it will take somewhat longer to arrive at the final end moments. However, the additional time consumed may be far more than equally repaid by the fact that the method is practically self-checking. Furthermore, upon completion of the analysis, one has at hand all necessary data for sketching, rapidly and accurately, the deflected structure, a procedure that experience has proved to be a most valuable tool both in analysis and in design. The method of balancing angle changes is particularly useful for studying secondary stresses in trusses and for investigating the effect of elastic distortion or of slip in the riveted or welded connections of a continuous steel frame. Approximate influence lines for live load studies can be obtained very rapidly thereby. The procedure of balancing angle changes is susceptible to expression in many forms. It is thought, however, that discussion will be used primarily for presenting evidence tending to test the theory proposed herein.

Introduction

Slope and Deflection.—The method of balancing angle changes should appeal to engineers who are most familiar with the slope-deflection equations

Note.—Discussion on this paper will be closed in December, 1936, *Proceedings*.

[1] Prof. of Structural Eng., Agricultural and Mechanical Coll. of Texas, College Station, Tex.

[2] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

because it makes use of the same physical variables of end slope, joint rotation, and relative deflection, at the same time possessing the other advantages which are so apparent in the method of moment distribution. There are other published methods[3] in which joint rotations have been obtained by series convergence, but these methods seem to involve relatively tedious procedures. The present method was developed by the writer in 1927. A few of the many special applications that have been found useful will be presented herein, but the basic method is essentially in its original form.

The well-known slope-deflection equations make it possible for one to compute the end moments in any unloaded member of a continuous frame from known values of end slopes and relative end deflections. If the member is loaded, the slope-deflection moments become the changes in the end moments from an originally fixed-end condition. Hence, the analysis of a continuous frame can be made upon the basis of a treatment in which joint rotations and relative deflections are chosen as the redundants. Many advantages have been claimed for the use of these functions as the redundants, most of which have not been justified by experience, but there is one which can not be disputed. Joint rotations and deflections are the equivalents of the physical action of the structure and the emphasis of these factors lends a desirable pictorial clearness to the method, provided this picture is not obscured by the mechanical details of the procedure.

Successive Rotation.—The method of balancing fixed-end moments starts with all members restrained against end rotation. The alternative procedure to be developed will start with all horizontal members acting as simply supported beams and all columns as pin-ended. Then, by successively balancing the angle changes at the joints until continuity is fully restored, one will obtain values of the true joint rotations from which the deflected structure can be sketched and the joint moments determined. Naturally, joint translation will complicate the analysis somewhat. The discussion that follows will present the method for use in continuous-beam analyses, after which its application to frames in which the joints translate will be considered briefly. A thorough understanding of a previous paper by the writer[4], will assist the reader in his interpretation of this paper.

Definition of Terms

A few new terms will be needed and for clearness they will be defined at once:

(1) By "angle change" (ϕ-value) will be meant the change in slope produced at the end of a member either by the loads or by an applied end moment; for example, certain easily determinable "angle changes", ϕ, occur at the ends of a simple beam when it is loaded.

[3] "Wind Stresses by Slope-Deflection and Converging Approximations", by J. E. Goldberg, Jun. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 962; also, "Rigid Frame Analysis by the Criterion Series of Converging Angular Approximations", by S. M. Cotten, M. Am. Soc. C. E. (Mimeographed notes under date of 1930.)

[4] "Wind Stress Analysis Simplified", by L. E. Grinter, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 610.

(2) By "joint rotation" (θ-value) is meant the angular twist of a joint in a loaded frame when absolute continuity exists between all members meeting at the joint.

(3) By "joint rotation factor" is meant the ratio between the joint rotation and the angle change existing at the end of a connecting member when the angular discontinuity between that member and the joint is removed by applied moments. It is assumed that the far ends of all members are pin-connected when the joint rotation factor is computed. Evidently, there is a joint rotation factor associated with each member meeting at a joint.

(4) By "carry-over factor" is meant the angle change at the far end of a simply supported or pin-ended member when the near end is rotated through an angle change of unity.

Procedure of Balancing Angle Changes

The condensed statement of the procedure of balancing angle changes that follows will be helpful to the reader, and must clarify itself, if it is used for continual reference while the illustrative examples are being studied.

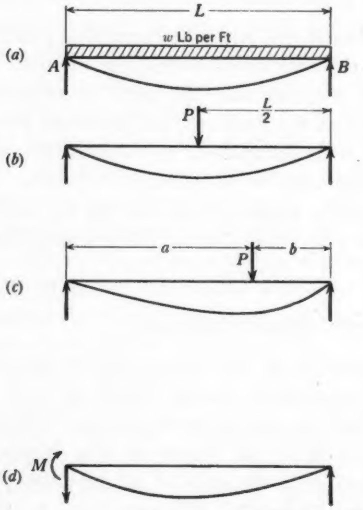
Angle Change and Joint Rotation.—With all joints of the frame held fixed against all possible translations and with the ends of all members either simply supported or pin-connected, one "permits" the members to deflect under the loads and computes the values of the end angle changes. Any joint is selected and the end angle change existing at that joint in one of its members is eliminated by twisting that particular member and the joint in opposite directions until continuity is restored. The recorded value of the joint rotation is the angle change that existed at the end of the member selected, after being multiplied by the joint rotation factor for that member.

Carry-Over Angles.—All members meeting at this joint have now been rotated through determinable end angles. The end angular rotation is the joint rotation for all members other than the one operated upon. For that particular member, the end angular rotation is the difference between the joint rotation and the end angle change caused by the loads. Since all these members were considered to be simply supported at their far ends, one must record an angle change (not a joint rotation) at the far end of each member equal to the angular rotation at the near end, when multiplied by the carry-over factor. Then, one proceeds from member to member and from joint to joint balancing the angles at each joint as many times as necessary, which means to continue the procedure until the carry-over angles are negligibly small. Continuity has then been established. Under normal conditions, it will be necessary to balance each joint from three to four times.

Simple Beam Angle Changes

The starting point in the analysis is to obtain the angle changes or the slopes at the ends of the loaded spans. This problem is solved in all text-books on the strength of materials. For rapid calculation one should use the

conjugate-beam method[5]. It scarcely seems necessary to present this treatment and instead the angle changes are given for a few simple cases in Fig. 1.



$$\phi_A = -\phi_B = \frac{w L^3}{24 E I}$$

$$\phi_A = -\phi_B = \frac{P L^2}{16 E I}$$

$$\phi_A = + \frac{P a b}{2 E I L}\left(\frac{2 b}{3} + \frac{a}{3}\right)$$

$$\phi_B = - \frac{P a b}{2 E I L}\left(\frac{2 a}{3} + \frac{b}{3}\right)$$

$$\phi_A = + \frac{M L}{3 E I}$$

$$\phi_B = - \frac{M L}{6 E I}$$

FIG. 1.—END ANGLE CHANGES FOR SIMPLE BEAMS.

JOINT ROTATION AND CARRY-OVER FACTORS

Stiffness of Joint and Member.—The stiffness of a beam may be defined as the end moment necessary to produce a unit end rotation and, therefore, it is proportional to the ratio, $\frac{I}{L}$, which is commonly designated as K. If an angle change, ϕ_1, occurring at the end of a member which frames into a joint, is to be eliminated and continuity is to be restored by twisting the member and the joint in opposite directions (by equal end moments), it is apparent from the definition of stiffness that the joint must be rotated through an angle, θ_1, equal to the angle, ϕ_1, times the stiffness ratio, $\frac{K_1}{\Sigma K}$. Here, as elsewhere, K_1 is the stiffness factor for the member having the end slope, ϕ_1, and ΣK is the sum of the stiffness factors for all members meeting at the joint. Hence, $\frac{K_1}{\Sigma K}$ is the "joint rotation factor" for Member No. 1 at the joint under consideration. In this paper, the far ends of all members are considered to be pin-connected or simply supported.

Usual Carry-Over Factor.—From Fig. 1 it is observed that the end angle changes in a simple beam loaded with a single end moment are in the ratio of 2 to 1. This represents the condition of a member when one end is being rotated into continuity. Hence, the carry-over factor for angle change is 0.5 whenever the member is prismatic or of uniform section.

[5] *Transactions*, Am. Soc. C. E., Vol. 98 (1927), p. 1.

Sign Convention

The simplest and most obvious sign convention is to call clockwise rotation positive. It follows that the end angle changes in a simply supported beam with downward loads will always be positive at the left and negative at the right. Furthermore, the same signs of angle changes apply for the case of a simple beam with a clockwise moment applied to the left end, as shown in Fig. 1(d). Accordingly, it follows that the carry-over factor for angle change is of negative sign, namely, $-$ 0.5.

The sign of a joint rotation, θ, necessarily is the same as that of the end angle change, ϕ, which produces it. Note, however, that the end of the member being operated upon rotates in the opposite direction through an angle, $\theta - \phi$, which determines both the value and the sign of the carry-over angle to the far end of this member. All of the carry-over angles to the far ends of the other members meeting at the joint are simply $-0.5\ \theta$.

Illustrative Example

The Frame.—Fig. 2 shows the application of the method of balancing angle changes to a continuous frame with prismatic members when all joints

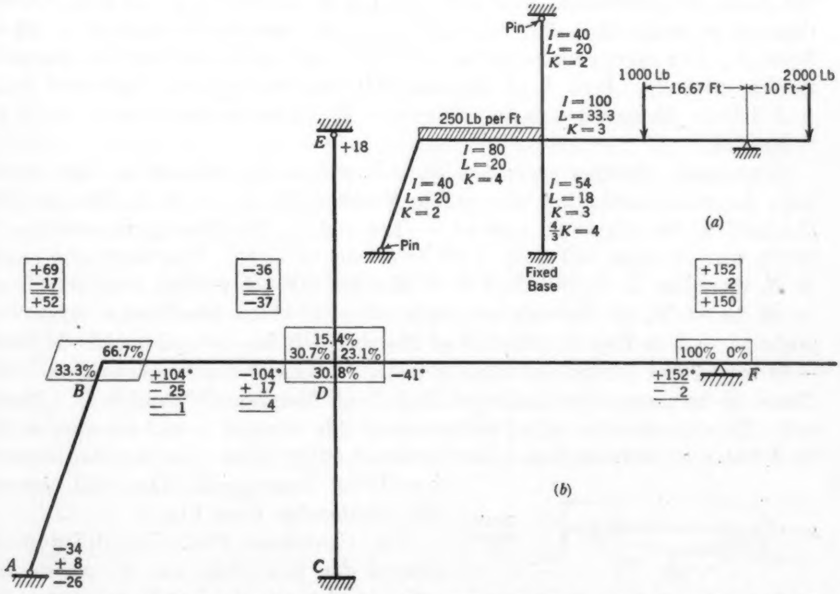


Fig. 2.—Balancing Angle Changes for Continuous Frame. (* = Relative Simple Beam Angle Changes in Loaded Spans; ϕ - Values.)

are fixed against translation. This academic example was chosen to illustrate, quite simply, the general procedure. Fig. 2(a) shows the loaded frame and gives the properties of the members. The end angle changes of the loaded spans, when the joints are pin-connected, are marked with an asterisk in

Fig. 2(*b*) which further illustrates the balancing of these angle changes to obtain the joint rotations. The end angle changes in Span *BD* are those of a simple beam carrying a uniform vertical load. The angle changes in the span, *DF*, are the resultant end slopes of the simple beam with a concentrated center load of 1 000 lb and an applied moment at the right-hand end of 20 000 ft-lb. From these joint rotations, the deflected structure can be sketched and the end moments calculated by use of the standard slope-deflection equations. Only the balancing of angle changes to obtain joint rotations will be discussed, since the remainder of the procedure should be clear to any one who is familiar with the ordinary theories of indeterminate structures.

Balancing the Angle Changes.—The balancing procedure was started (Fig. 2(*b*)) at Joint *B*, although any other unbalanced joint might have been selected. The angle change of + 104 at End *B* of Member *BD* produced a joint rotation of 66.7% of + 104 or + 69. This is simply the angle change times the joint rotation factor, or $\phi\left(\frac{K_1}{\Sigma K}\right)$. Joint rotation factors $\left(\frac{K}{\Sigma K}\text{-values}\right)$ are expressed as percentage factors and enclosed in boxes at the joints for convenient reference. End *B* of Member *BA* has been rotated through an angle of + 69 which gives rise to a carry-over angle of − 34 to Joint *A*. The carry-over factor is − 0.5 and only whole numbers are recorded in this example. End *B* of Member *BD* has been rotated backward from + 104 to + 69, or through an angle of − 35. The carry-over angle is + 17 to End *D*.

Two angle changes occur at Joint *D* which was selected as the second joint to be balanced; that is, − 41 in Member *DF* and − 87 in Member *DB* obtained as the algebraic sum of − 104 and + 17. The joint rotation is 23.1% of − 41 plus 30.7% of − 87, or a total of − 36. The carry-over angle to *E*, therefore, is + 18. End *D* of Member *DF* has rotated positively from − 41 to − 36, or through an angle of + 5 which produces a carry-over angle of − 2 to End *F*. End *D* of Member *DB* has rotated positively from − 87 to − 36, or through an angle of + 51. The carry-over angle to *B* is − 25. There is no carry-over angle to End *C* of Member *DC* which is a fixed end. To allow for the added resistance of this member to end rotation at *D*, its *K*-value or stiffness factor was increased $33\frac{1}{3}$% when joint rotation factors were being determined. One will observe this relationship from Fig. 3.

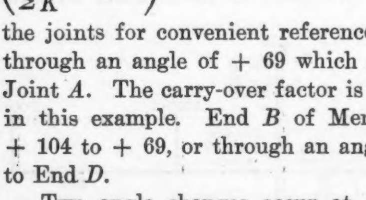


FIG. 3.—EFFECT OF END RESTRAINT UPON STIFFNESS.

The Cantilever End.—The third joint selected for balancing was *F*, where the entire resistance to rotation occurs in Member *FD* which has a joint rotation factor of 100 per cent. Hence, each value of ϕ is simply recorded as a joint rotation, θ, and there is no carry-over angle to Joint *D*. In other words, there is no external resistance to rotation at Joint *F* which means that it can be treated like Joints *A* and *E* where there is no distinction

made between values of ϕ and θ. The fourth and fifth joints balanced were B and D which involved merely a repetition of the procedure that has already been described. The repetition was necessary in order to balance out the carry-over angles mentioned previously.

Accuracy in Balancing Angles

In Fig. 2 the balancing process was stopped when the carry-over angles became less than unity. This is the proper procedure. Greater accuracy could be obtained if desired by introducing an extra significant figure into the original angle changes or ϕ-values. A device frequently found useful is to double or triple the first three significant figures of the original ϕ-values and then balance joints until the carry-over angles are less than unity. This device can be used to improve the accuracy of the analysis without adding a fourth significant figure to the original ϕ-values. Of course, the final joint rotations must be reduced in the same ratio that the original angle changes were increased. Accuracy was sacrificed for clearness in Fig. 2.

Checking the Final Moments

The simplest procedure for checking the analysis is to apply the equation $\Sigma M = 0$ to each joint after final moments are determined from the θ-values by use of the slope-deflection equations. Since the balancing is performed on angles rather than moments and since fixed-end moments are introduced through the slope-deflection equations, any error in the computation of an original angle change, in the balancing procedure, or in the application of the slope-deflection equations will be detected by this simple check. Of course, compensating errors might occur, but this slight possibility for error exists in all methods. A sketch of the deflected structure will help to locate such unusual errors.

Haunched Beams and Curved Members

Basic Terms Redefined.—A variation of cross-section or a curved axis affects the resistance of a member to end rotation and makes necessary a careful calculation of end angle changes, joint rotation factors, and carry-over factors. End angle changes can be computed conveniently by use of the conjugate-beam method. Since the possible variations of loading are infinite, it would be unreasonable to attempt to formularize this procedure. Joint

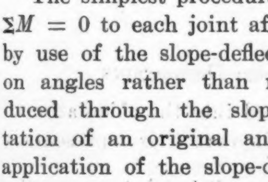


Fig. 4.—Stiffness of a Haunched Beam.

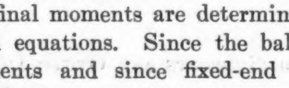


Fig. 5.—Settlement of a Support.

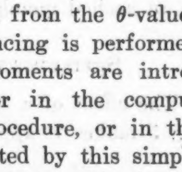


Fig. 6.—Rotation of a Footing.

rotation factors and carry-over factors can be obtained quite simply from the angle changes that occur at the ends of the simply supported beam loaded with a unit moment at one end (see Fig. 4). The carry-over factor for angle

change from Point A to Point B is $\frac{\phi_B}{\phi_A}$ and has a negative sign as in a prismatic beam. Stiffness is defined rigorously as the moment necessary to produce a unit end rotation, and evidently is $\frac{1}{\phi_A}$. The factor, K, has been used for prismatic beams to define relative stiffness. Hence, since $\phi_A = \frac{L}{3\,E\,I} = \frac{1}{3\,E\,K}$, or since $\frac{1}{\phi_A} = 3\,E\,K$ for such a beam, it is permissible and possibly desirable to omit the factor, $3\,E$, in making calculations for a haunched beam. Accordingly, one can define K as $\frac{1}{3\,E\,\phi_A}$ for the haunched beam and then the definition for the joint rotation factor remains unchanged as $\frac{K}{\Sigma\,K}$. Curved members can be handled by this procedure without re-defining any terms. The carry-over factor for angle change may reverse its sign for an arched member.

Settlement of Supports and Other Impressed Distortions

The effect of a settling support is readily introduced into the calculations. Fig. 5 makes it apparent that a settlement of Δ at Support C is represented by angle changes of $\phi_1 = + \frac{\Delta}{L_1}$ at the two ends of Span BC and $\phi_2 = - \frac{\Delta}{L_2}$ at each end of Span CD. One balances these angle changes and computes the moments in exactly the same manner as for angle changes produced by loads. There are no fixed-end moments to be considered when applying the slope-deflection equations. Temperature change of a continuous frame gives rise to linear distortions represented by end angle changes in a similar manner. The unplanned rotation of a footing could be represented by two angle changes, as shown in Fig. 6.

Influence Lines for Continuous Beams

The Deflected Load Line.—The process of balancing angle changes offers a highly pictorial method of determining influence lines for continuous beams. It is well known that the influence line for any function, such as moment or shear, can be obtained as the deflected load line produced by an impressed unit distortion in the nature of the stress function; that is, a unit rotation must be impressed upon the structure to obtain an influence line for moment and a unit shearing distortion must be used to obtain an influence line for shear. The corresponding end angle changes are illustrated in Fig. 7. Fig. 7(a) shows the angle changes that will give rise to an influence line for M_C, the moment over Support C. In balancing angles, one maintains the discontinuity, $\phi = 1$, at Joint C by balancing only carry-over angles at Joint C.

An influence line for moment between supports of a continuous beam would be obtained as illustrated by Fig. 7(*b*). In this case the end angle changes, $\phi_1 = -\dfrac{b}{a+b}$ and $\phi_2 = +\dfrac{a}{a+b}$, must be balanced out and the

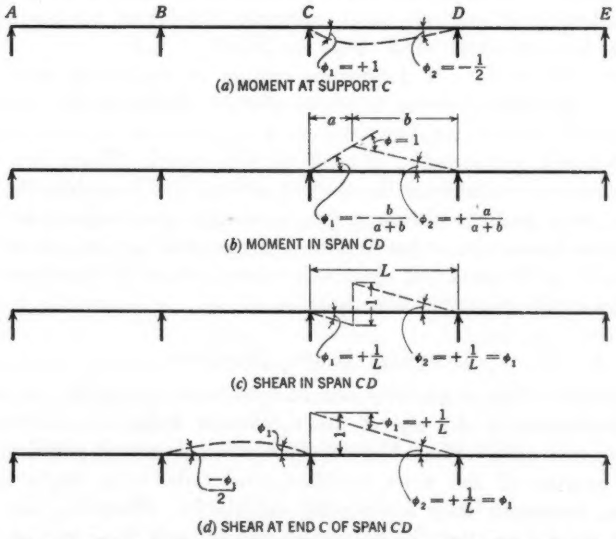


FIG. 7.—ANGLE CHANGES FOR PRODUCING INFLUENCE LINES FOR A CONTINUOUS BEAM

internal, unit, angular discontinuity will be maintained automatically by the standard procedure of balancing angles. Similar configurations for obtaining influence lines for shear between supports and at a support, are illustrated by Fig. 7(*c*) and Fig. 7(*d*). Observe that there are no angular discontinuities to be removed at the sections of vertical displacement.

Approximate Influence Lines.—The procedure, as described for continuous beams, is quite as applicable to all types of continuous frames in which joints do not translate. Hence, the method may be particularly useful as a tool for sketching an influence line rapidly and for obtaining the approximate load divides; that is, balance the angle changes shown in Fig. 7 without making any attempt at great accuracy, and then, from the approximate values of the joint rotations, sketch the shape of the deflected structure, which is the influence line desired. Approximate influence lines are frequently quite as useful as exact ones. However, the exact influence lines can also be obtained rapidly by this method.

THE PROBLEM OF JOINT TRANSLATION

Known or Estimated Deflections.—Rigid frame structures usually have certain joints that are free to translate as permitted by the elastic distortions of the structure. Evidently, the analysis of such a structure would be simplified by a previous knowledge of the deflections. From the known deflections,

it would be possible to write a corresponding set of angle changes, balance these angle changes to obtain the joint rotations, and then compute the corresponding end moments. When exact deflections cannot be known beforehand, it usually is possible to estimate the deflections approximately and to base the analysis upon them, later introducing correction factors if necessary. The procedure of basing an analysis upon estimated deflections has been presented by the writer in a paper on wind stress analysis.[4]

Successive Distortion.—If deflections cannot be estimated with sufficient accuracy, one can always revert to the device of obtaining the shears in all stories or panels from an applied distortion (represented by equal end angle changes) produced, successively, in each story or panel. Then, by combining these configurations to produce the desired shears, one has obtained the true deflected structure and the corresponding moments are precise. This process is more exact and consumes more time than is justified in wind stress analysis, but it is useful in the study of open web trusses where it furnishes the data required for a study of partial live loadings.

Secondary Stress Analysis

Bar Rotations.—The analysis of secondary stresses in bridge trusses forms a perfect illustration of the use of predetermined deflections for computing indeterminate moments. Since the deflections are produced primarily by the changes in lengths of the truss members and only to a slight extent by the secondary moments, they are readily calculable. However, only the bar rotations are needed to start the balancing process and these can be obtained without the formality of drawing a Williot-Mohr diagram. The angle changes of the triangles of a truss can be expressed by the following relations which were derived by the late Milo S. Ketchum, Hon. M. Am. Soc. C. E., in his book entitled "Steel Mill Buildings":

$$E\,(\Delta\, A) = (s_{bc} - s_{ab}) \cot B - (s_{ca} - s_{bc}) \cot C \ldots\ldots\ldots (1)$$

$$E\,(\Delta\, B) = (s_{ca} - s_{bc}) \cot C - (s_{ab} - s_{ca}) \cot A \ldots\ldots\ldots (2)$$

and,

$$E\,(\Delta\, C) = (s_{ab} - s_{ca}) \cot A - (s_{bc} - s_{ab}) \cot B \ldots\ldots\ldots (3)$$

In Equations (1), (2), and (3), ΔA represents the change in Angle A caused by the stresses, s_{ab}, s_{ca}, and s_{bc}, in the correspondingly lettered bars of the truss of Fig. 8. Accordingly, if it is assumed that Bar *a-c* remains horizontal, ΔA becomes the rotation of Bar *a-b*. Similarly, Bar *b-c* rotates through an angle, $\Delta A + \Delta B$, Bar *b-d* rotates through an angle, $\Delta A + \Delta B - \Delta D$, etc. These bar rotations are relative to a zero rotation for Bar *a-c*. The true rotations relative to a line through the supports, or *a-i*, are obtained by subtracting from each bar rotation relative to Bar *a-c* the true rotation of Bar *a-c*, which,

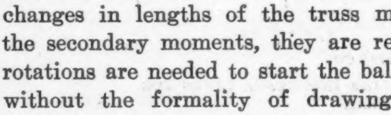


FIG. 8.—BAR ROTATIONS FOR SECONDARY STRESS ANALYSIS.

for Fig. 8, can be shown to be the average relative bar rotation for all lower chord members.

Secondary Moments.—Bar rotations are treated as end angle changes and balanced by the usual procedure to obtain joint rotations. Then, the secondary moments follow from the slope-deflection equation,

$$M_{AB} = 2\,E\,K\;(2\;\theta_A + \theta_B - 3\,\rho) \quad \text{.................(4)}$$

in which θ represents a joint rotation and ρ is the bar rotation.

Bent Analysis

Shear Adjustment by Proportion.—A simple bent will be used to illustrate the general method of adjusting shears by proportion. Such a bent is characterized by having a single degree of lateral freedom — its side lurch. Hence, if moments and shears are known for any given side-sway, those corresponding to any other side-sway can be obtained by multiplying the known moments and shears by the ratio of lateral deflections. It follows that the balancing procedure can be started by imposing upon the bent an unresisted side lurch represented by equal angle changes at the top and bottom of each column. Then, by balancing angle changes, one obtains the joint rotations from which the column shears, V_H, are determined by the relation:

$$V_H = \frac{6\,E\,K}{h}\;(\theta_A + \theta_B - 2\,\rho) \quad \text{.................(5)}$$

The ratio of actual horizontal shear to total calculated horizontal shear is then to be used as a multiplication factor for adjusting lateral deflection, joint rotations, and column and girder moments. The "actual" horizontal shear may be caused by lateral loads or by an artificial restraint introduced through the balancing process when one is studying the effect of dissymmetry.

Analysis Involving Joint Slip

Riveted connections undergo elastic distortion and slip, both of which permit angular discontinuities to occur at the joints. For the purpose of outlining a method of analysis, it is unnecessary to distinguish between elastic and plastic joint deformation, although it is evident that these effects would have to be isolated and studied separately to investigate the action of moving loads.

Balancing Moments and Angles.—Test data on joint slip are available in a paper[6] by J. Charles Rathbun, M. Am. Soc. C. E. The cases to be discussed will be chosen so that these data are applicable. The method of analysis given herein will involve the double conception of balancing moments and balancing angle changes. The procedure can be developed entirely in terms of balancing moments, but the combination suggested herein has the advantage of permitting the use of standard stiffness factors and the unchanged carry-over factor of 0.5 for a prismatic member. The method

[6] *Proceedings*, Am. Soc. C. E., January, 1935, p. 3.

of balancing angle changes can also be used without the aid of balancing moments but less advantageously.

Continuous Beams with One Elastic Connection.—The first example presented (Fig. 9) is one solved by Professor Rathbun[7]. Joint B is assumed to

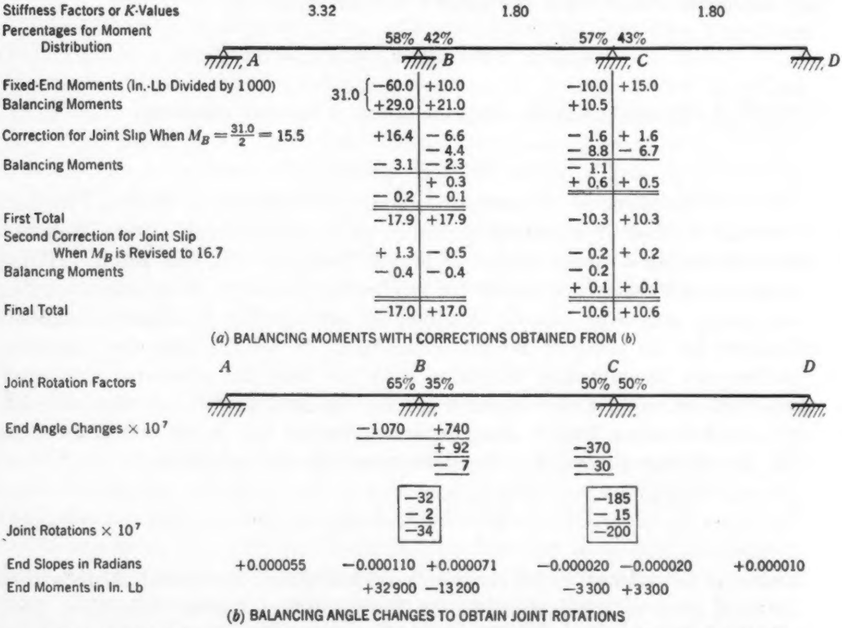


FIG. 9.—ANALYSIS OF CONTINUOUS BEAM WITH ELASTIC CONNECTION AT JOINT B.

be an elastic connection having a modulus of rotation of 4.2×10^8 in-lb per unit (radian) rotation in Span BC and 2.9×10^8 in-lb per unit rotation in Span BA.

Fixed-end moments are computed and the moments at Joint B are balanced once to obtain the first approximation of 31 000 in-lb for M_B. This moment will be reduced considerably by joint deformation and as a starting point it was assumed that the final moment at Joint B would be reduced 50%, to 15 500 in-lb. The moment at Joint B gives rise to angle changes from joint deformation of $\phi_{BA} = \dfrac{31\,000}{290\,000\,000} = 0.000107$; and, $\phi_{BC} = \dfrac{31\,000}{420\,000\,000} = 0.000074$. These angle changes must be maintained as angular discontinuities and, accordingly, they are not balanced. Nevertheless, there is a carry-over angle of $-0.5\ \phi_{BC}$ to Joint C which is to be balanced as in previous examples. (See Fig. 9(*b*).)

End moments are calculated from end rotations by the slope-deflection equations and are introduced as fixed-end moments. Joint moments are then completely balanced, and the joint moment at B is found to be 17 900 in-lb

[7] *Proceedings*, Am. Soc. C. E., January, 1935, p. 40, Fig. 33.

as compared to the estimated moment of 15 500 in-lb. The true moment lies between these values and was then estimated at 16 700 in-lb. Correction moments can be obtained in this simple example by proportion since there is only one elastic joint in the structure. The revised value of M_B is 17 000 in-lb, which is close enough to the assumed value of 16 700 in-lb to make a further

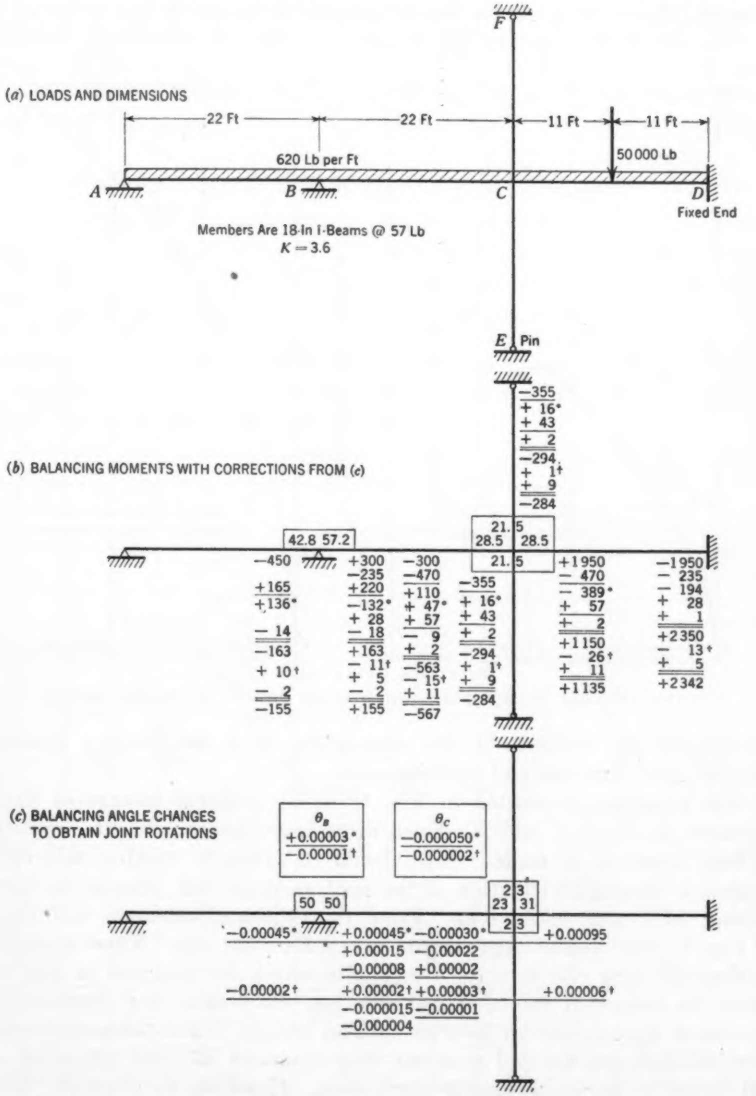


FIG. 10.—ANALYSIS OF A FRAME WHERE JOINT SLIP IS INVOLVED. (*CORRECTIONS FOR JOINT SLIP ARE BASED ON ASSUMED MOMENTS OF:—$M_{BA} = M_{BC} = +150$ INCH-KIPS; $M_{CB} = -600$ INCH-KIPS; AND $M_{CD} = 1100$ INCH-KIPS. † REVISION OF CORRECTIONS FOR JOINT SLIP WHEN:—$M_{BA} = M_{BC} = 155$ INCH-KIPS; $M_{CB} = -570$ INCH-KIPS; AND $M_{CD} = 1140$ INCH-KIPS.)

correction unnecessary. These final moments check the values computed by the writer by Professor Rathbun's procedure.

Continuous Beam with Two Deformable Connections.—The second example, shown in Fig. 10, represents a more complicated analysis of a frame having two joints where both elastic deformation and plastic joint slip occur. The modulus of rotation for the connections at Joints B and C are as shown by Fig. 11. It is of importance to note that it is not permissible in this case

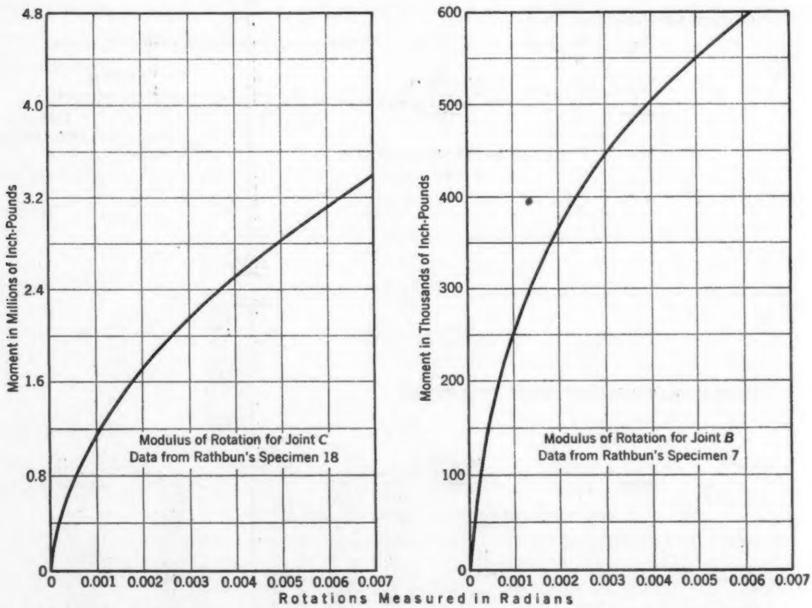


FIG. 11.—MODULI OF ROTATION FOR RIVETED JOINTS OF FRAME OF FIG. 10.

to simplify the analysis by the assumption of a straight-line relationship between joint slip and end moment.

The procedure is started in Fig. 10(*b*) by a single balance of fixed-end moments at Joints C and B. From these approximate moments, an estimate of final moments is made. Care should be taken in making this estimate because a close approximation of the final moments will remove the need for making additional corrections. From the estimated moments and the data of Fig. 11, one obtains approximate values for joint slip. These angular discontinuities give rise to carry-over angles which are balanced in Fig. 10(*c*). From the computed values of end rotation, one obtains end moments which are shown followed by an asterisk in Fig. 10(*b*). When these moments had been balanced out, the end moments were compared with the estimated values and found to be in reasonable agreement. However, to illustrate the procedure of introducing corrections, an improved estimate of final moments was made, and the changes in these moments were used to obtain correction angles from Fig. 11, which also are balanced in Fig. 10(*c*). The correc-

tion moments obtained from these correction angles are introduced into Fig. 10(*b*) and balanced to obtain final moments. Second corrections to moments and angles are marked with a dagger in Fig. 10(*b*) and Fig. 10(*c*).

Sign Reversal.—The foregoing procedure will not be found easy to understand until one is quite familiar with the methods of balancing moments and balancing angle changes. Possibly a confusing point is the reversal of signs of the moments as obtained by the slope-deflection equations. This is necessary because of the fact that the signs of internal moments are used in the balancing procedure, whereas the signs obtained by the standard application of the slope-deflection equations apply to the external resisting moments.

Direct Method of Distributing Joint Rotations

The conception of balancing angle changes can be extended into a direct method of distributing joint rotations that is useful where only a single span of an elastic frame is loaded. It is also valuable as a tool for rapidly obtaining approximate influence values; for instance, the joint rotations through the frame produced by a unit angle change (ϕ-value) at one end of a particular member. The extent to which this method of direct distribution might take the place of the general method of balancing angle changes depends only upon the individual preference of the reader.

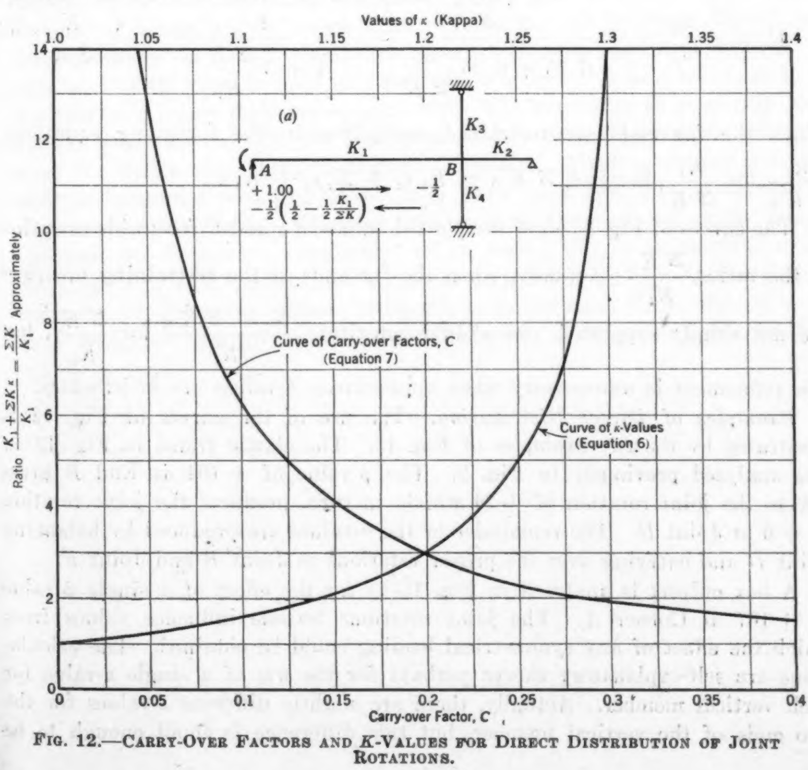


FIG. 12.—CARRY-OVER FACTORS AND *K*-VALUES FOR DIRECT DISTRIBUTION OF JOINT ROTATIONS.

Stiffness Dependent Upon End Restraint.—Fig. 12(*a*) shows the resultant θ-values as produced by a unit ϕ-value at End A of Member AB which is restrained but not fixed at Joint B. Since stiffness is defined as resistance to rotation, the stiffness of the member in terms of rotation at End A is increased by the restraint at Joint B. The stiffness at a joint usually is expressed as $\Sigma K = K_1 + K_2 + K_3 + K_4$. If κ is defined as the ratio of the stiffness of the restrained member to its stiffness when the far end, B, is simply supported, then $K\kappa$ becomes the stiffness factor for a restrained member exactly comparable to the K-value for an unrestrained member. From the small sketch, Fig. 12(*a*), it will be evident that $\theta_A = 0.75 + 0.25 \frac{K_1}{\Sigma K}$ and that $\theta_B = -0.5 \frac{K_1}{\Sigma K}$. Accordingly,

$$\kappa = -\frac{1}{\theta_A} = \frac{1}{0.75 + 0.25 \frac{K_1}{\Sigma K}} = \frac{4}{3 + \frac{K_1}{\Sigma K}} \quad \text{..........(6)}$$

The carry-over factor for joint rotation is,

$$C = -\frac{\theta_B}{\theta_A} = -\frac{0.5 \frac{K_1}{\Sigma K}}{0.75 + 0.25 \frac{K_1}{\Sigma K}} = -\frac{2}{1 + 3 \frac{\Sigma K}{K_1}} = -\frac{\kappa}{2} \frac{K}{\Sigma K} \quad .(7)$$

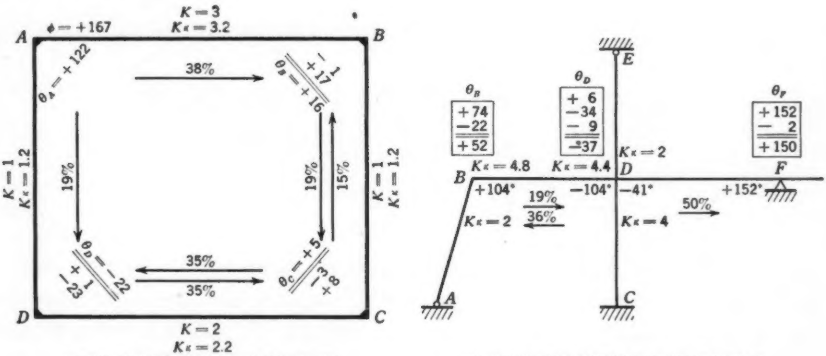
When the far ends are restrained, substitute in the foregoing equations, $\frac{K_1}{\Sigma K \kappa}$ for $\frac{K_1}{\Sigma K}$, in which $\Sigma K \kappa = K_2 \kappa_2 + K_3 \kappa_3 + K_4 \kappa_4$.

The curves of Fig. 12 show the plotted values of κ and C for a wide variation of the ratios, $\frac{\Sigma K}{K_1}$. Actually, when the far ends of the restraining members are not simply supported, one should substitute $\frac{K_1 + \Sigma K \kappa}{K_1}$ for $\frac{\Sigma K}{K_1}$, but this refinement is unnecessary when approximate θ-values are satisfactory.

Examples of Direct Distribution.—The use of the curves of Fig. 12 is illustrated by the two examples of Fig. 13. The elastic frame in Fig. 13(*b*) was analyzed previously in Fig. 2. The ϕ-value of + 104 at End B gives rise to the joint rotation of + 74 which, in turn, produces the joint rotation of + 6 at Joint D. The remainder of the θ-values are produced by balancing Joint D and carrying over the proper rotations to Joint B and Joint F.

A box culvert is analyzed in Fig. 13(*a*) for the effect of a single ϕ-value of + 167 at Corner A. The joint rotations become influence values from which the effect of any symmetrical loading could be obtained. The calculations are self-explanatory except perhaps for the use of a single κ-value for each vertical member. Actually, there are slightly different κ-values for the two ends of the vertical member, but this difference is small enough to be

neglected. The corresponding effect upon the carry-over factor was of greater importance, and, accordingly, two carry-over factors were recorded for the vertical member.



(a) INFLUENCE VALUES FOR CLOSED FRAME (b) ANALYSIS OF CONTINUOUS FRAME IN FIG. 2

FIG. 13.—DIRECT DISTRIBUTION OF JOINT ROTATIONS. (*RELATIVE ϕ-VALUES.)

CONCLUSION

The procedure of balancing angle changes has been found to offer a parallel method to that of balancing moments. Either tool can be used to analyze continuous beams, rigid frames, open-web trusses, tall building frames, continuous arches on elastic piers, and other continuous framed structures. In certain cases they can be used to advantage to supplement each other, as in the analysis of a frame undergoing joint slip. The procedure of balancing angle changes possesses the inherent disadvantage that final moments are computed from the differences between angular rotations. Hence, angular rotations must be computed with greater accuracy than joint moments by the method of moment distribution. However, the method of balancing angle changes possesses two important advantages of its own: (1) It is essentially self-checking; and (2) it offers one the clearest possible picture of the physical action of the structure. These attributes seem sufficient to justify its use by those designers who are accustomed to thinking in terms of joint rotations and deflections.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

THE MODERN EXPRESS HIGHWAY

By Charles M. Noble,[1] Assoc. M. Am. Soc. C. E.

Synopsis

The purpose of the paper is to present for criticism and discussion a process of reasoning and analysis in highway design rather than to set forth definite rules and data; to suggest necessary test and research projects; to emphasize that design should provide positive safety at the speeds of which vehicles are now capable, or may be capable, in the near future; and to suggest an immediate raising of design standards for trunk highways.

The paper presents only an outline of a broad subject and is intended principally to direct attention in a positive manner to safety in highway design.

Introduction

The enormous loss of life and property on American streets and highways is a compelling reason for pausing to take cognizance of location and design

TABLE 1.—Types of Accidents Resulting in Persons Killed and Injured in 1935

Collision with	Accidents		Persons Killed		Persons Injured	
	No.	Percentage of total	No.	Percentage of total	No.	Percentage of total
Pedestrian	297 610	36.0	16 030	44.4	276 640	30.9
Automobile	374 490	45.3	8 900	24.6	450 320	50.3
Horse-drawn vehicle	4 960	0.6	140	0.4	5 370	0.6
Railroad train	4 960	0.6	1 440	4.0	4 480	0.5
Street car	13 230	1.6	310	0.9	11 640	1.3
Other vehicle	8 270	1.0	250	0.7	8 060	0.9
Fixed object	53 730	6.5	4 080	11.3	64 460	7.2
Bicycle	19 840	2.4	580	1.6	17 910	2.0
Non-collision	47 120	5.7	4 290	11.9	53 720	6.0
Miscellaneous	2 480	0.3	80	0.2	2 680	0.3
Total	826 690	100.0	36 100	100.0	895 280	100.0

practice and of the manner in which it should be improved to meet the increased speed of the present and future motor car. No other engineering structure has such a death record as the highway. Table 1 is a compilation

Note.—Discussion on this paper will be closed in December, 1936, *Proceedings*.

[1] Asst. Engr., Port of New York Authority, New York, N. Y.

of the accident record in the United States for 1935. Certainly many more accidents are chargeable to the highway itself than statistics indicate. The failure to attribute accidents to faulty road design is due to several causes principal of which is the subtlety of the problem. Police officials are not always trained to analyze the basic causes of accidents properly, and there is a definite temptation to blame the driver because such a course is more concrete and along lines of least resistance.

The motor-car designer has set a pace which the highway designer has not anticipated properly. New automobile models appear each year, but new highways to replace outmoded highways cannot be produced so quickly. This imposes on the designer the task of producing a highway suitable for the car of ten or fifteen years in the future. So far, in general, this has not been done. The result is a staggering economic loss in life, property, and capital expenditure. To-day (1936) there is a large road mileage, less than ten years old, which is hopelessly obsolete so far as the safe and efficient operation of the present motor car is concerned. As an example, in 1935 several manufacturers of motor vehicles (some cars in the medium-price class) advertised that their cars would maintain a sustained speed of 100 miles per hr. The roads to accommodate these cars are not now in existence.

The rise in the accident and death rate is recognized to be due to increased speeds. The vehicle itself is far safer than formerly, but the higher speeds result in more serious accidents, involving more cars with a consequent greater loss of life. At high speed there is less time to correct an error of judgment, and it must be recognized that this time element is the new factor of importance which has entered the problem. Therefore, basic safety must be designed into the highway, taking this factor into account in such manner that the motorist is allowed sufficient time to cope with driving hazards. The reckless and unsafe driver must be considered. He affects the life and property of the innocent safe driver and cannot be disregarded with a shrug as formerly. In the past emphasis in design has been based on highway economics founded on vehicle operating costs and time losses. Broad highway economics founded on loss of life and property, as well as operating costs, has not received much attention. The injuries and the loss of life and property on American highways are issues squarely facing the highway engineer. It is his duty to design the highway so that the traveling public is safeguarded.

Fundamental Concepts

A consciousness of driver psychology is gradually emerging although a knowledge of its "laws" is only vague and half-formed. More knowledge is necessary, but, in the meantime, there should be a consciousness of its existence, and such facts as are now known should be applied to design. For instance, in many cases, it is possible to design a highway so that the motorist is led unconsciously to choose the safe act rather than that which is unsafe.

The trend of the times indicates the desirability of designing the highway on the basis of a constant rate of vehicle speed between large terminal points.

It is evident that the decision as to the design speed should be arrived at only after very thorough study of the country to be traversed and that it should be made personally by the principal officer in authority. In the interest of safety it is also evident that the motorist must be notified on any change in the design speed. From present indications it would appear not unreasonable to design for a speed of 100 miles per hr in country not too mountainous. This will result in trunk highways having better alignment and curvature than the high-speed passenger railways. Generally, the present railway maximum speed is approximately 80 miles per hr. The drivers of trains are trained, picked men, familiar with their runs, riding on fixed rails, with definite orders and under close supervision. The reverse is true on the highways.

At every step in design the question should be asked: Is this safe under all driving conditions—darkness, snow, ice, rain, fog, ice and snow on the pavement, snow plowed on to the shoulders? Every conceivable accident hazard must be analyzed. Always there should be the consciousness of the number of feet per second a vehicle travels at any given speed, the length of headlight beams, the chances of visibility, all combined with the distance required to bring a vehicle to a stop.

The Dual Highway

The dual highway appears to be the only type of design developed so far to accommodate successfully the present high speeds with safety. A suggested minimum design is shown in Fig. 1. To provide increased speed with safety, wider lanes will be necessary. It is suggested that two 12-ft lanes in each direction may be sufficient if truck traffic is relatively light.

Fig. 1.—Suggested Minimum Dual Highway Design

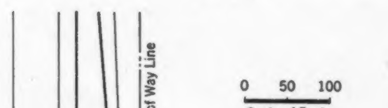


Fig. 2.—One Quadrant of Suggested Access Road Design; Other Three Quadrants Are Similar

In the case of heavy truck traffic two lanes in each direction may be constructed in the unused center portion of the dual highway, but without any physical connection with the high-speed lanes. Access to the truck lanes may be provided at intervals by the use of "fly-under" crossings beneath (or over) the high-speed lanes. It does not appear likely that trucks will travel much in excess of 70 miles per hr and, therefore, the standards of design for these center lanes, except for ruling grades, need not be as high as for the outer lanes. A most effective method of eliminating time losses is to by-pass all towns and cities between terminals, in addition to separating all highway and railway grade crossings. A safety factor is to prevent frontage of any type on the highway and to exclude farm and local road entry. It appears reasonable to restrict access to approximately 10-mile intervals.

Fig. 2 illustrates a simple type of design for rural access which has much to recommend it from a safety standpoint. The acceleration and deceleration lanes are the feature of the design. They provide full visibility on the trunk highway over a period of time sufficient to enable the motorist to adjust himself to the entrance and exit of other vehicles.

In the interest of economy the dual highway may be constructed in stages, but upon a full-width right of way. Two lanes may be constructed on one side and used as a two-way road until such time as funds become available for the complete project.

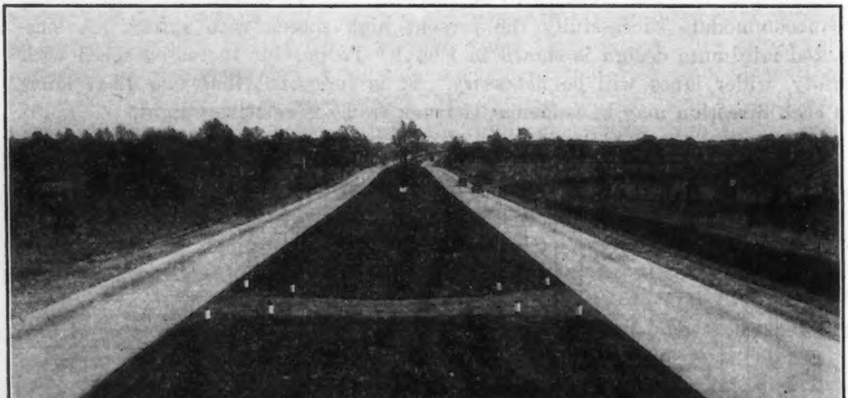


FIG. 3.—DUAL HIGHWAY DESIGN IN DELAWARE.

It is unnecessary to enumerate the points of superiority of the dual highway over conventional multiple-lane design to any one who has had the good fortune to drive over a properly designed dual highway. The elimination of the passing problem alone is sufficient reason for the adoption of this principle. Esthetically, also, it is capable of great possibilities and development. It is certain that the dual highway deserves very serious consideration in the present and future planning of trunk highways. Fig. 3 shows the dual highway as at present developed in Delaware.

Details of Design

In order to define more clearly the writer's views with respect to highway design it will be more illustrative if details of design are discussed separately under certain more or less specific sub-divisions.

Super-Elevation.—With present and probable future high speeds it seems untenable not to bank (super-elevate) curves. In many sections of the United States banking is standard practice, but in some States super-elevation has been abandoned on multiple-lane projects on the plea of drainage difficulties. There appears no good reason why effective means of meeting the situation cannot be developed. A proper study of drainage and suitable design can eliminate nearly all the accumulated water which would normally flow transversely across the pavement. Certainly the motorist has a right to expect the protection of super-elevation around curves. Negotiating at high speed a curve with "reverse" banking when the surface is covered with ice or snow is far from a pleasant or safe experience.

As far as the writer is aware the proper rate of banking for any given speed-radius combination is unknown. Comprehensive tests using modern passenger cars and trucks on clear and icy pavements is the logical approach to the solution of the problem. The formula in present use is,

$$E = 0.067 \frac{V^2}{r} \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

in which E equals super-elevation, in feet per foot of width; V equals velocity, in miles per hour; and r equals radius, in feet. Equation (1) yields rates somewhat higher than are required for practical driving conditions. It is based on the super-elevation maintaining equal weight on the inside and outside tires. Practically, this is not necessary, as demonstrated from experience the outside tires may have considerably more weight on them than the inside tires without danger. The point is that there are no quantitative data upon which conclusions can be based as to how much this unbalanced weight may be for safe, comfortable operation around curves. The present method of determining the super-elevation is to use Equation (1), but to assume some arbitrary speed lower than the actual speed expected. Here is an opportunity for much needed research.

Curves.—From an analytical standpoint the minimum radius will depend on the maximum permissible rate of banking and the speed assumed. For example, suppose it is desired to design for a speed of 100 miles per hr, assuming 1 in. per ft of width as the maximum rate of super-elevation and, further, that Equation (1) will bank the curve safely if V is taken as 60 miles per hr. Substituting in the formula, a minimum radius of 2 900 ft is indicated. This method of determining the minimum radius is superior to the present practice of arbitrarily setting up a standard minimum radius based upon judgment only.

It is doubtful whether spiral curves will be necessary. The vehicle is free to spiral the slight amount necessary within the wide lane in order to accommodate itself to the large radii required by present speeds. Opinion is not

of much value in solving this problem, however. Practical road tests are necessary before design practice can be established definitely.

Careful thought and designing are necessary in providing the transition between the normal crown and the banked section. It does not seem reasonable to adopt a standard length and treatment regardless of the radius and rate of banking. Certainly the treatment should be such that there is no tendency for the vehicle to be "thrown in" toward the center of the curve, or to swerve out. The ideal is for the design to be such that the vehicle may enter and leave the curve smoothly—almost automatically.

The distance between reversed curves depends on the time required for the driver of a vehicle to "come out" of one curve and prepare to enter the next one. As an illustration, if a 7-sec period and a speed of 100 miles per hr is assumed, the minimum distance between reversed curves will be approximately 1 000 ft. The writer is unaware of any observations upon which to base any conclusion as to the proper reaction time between reversed curves. This time interval may have some relation to the radii of the curves.

Grades.—The realization is growing that long smooth grades and vertical curves are essential for confident and safe operation at high speeds. Rolling grades create a hazard for night driving. Shadows cast by headlights result in lack of vision and loss of confidence on the part of the driver. A policy of encouraging night travel would tend to spread the traffic more evenly over the entire 24 hr, thus making a more efficient use of the investment in the highway. Long, sustained profiles, being more attractive to the night driver, may prove more economical than rolling grades when considered in the light of broad economics.

At a speed of 100 miles per hr a car travels 146 ft per sec. It would appear reasonable to provide uniform grade conditions for a period of at least 10 sec. If this speed is assumed in the design, a minimum length of profile tangent of 1 500 ft is indicated. The lengths of vertical curves should be such that, within the limit of vision of the headlights, shadows will not be cast and the normal length of the head-light beam will not be reduced. This will result in much longer vertical curves than those now in use. In general, definite data from which the proper length of vertical curves may be set are not available. Tests or optical computations will be necessary before definite values can be presented.

Present thought appears to favor good alignment in preference to low maximum grades, but it would seem desirable to limit maximum grades to about 5% in order to provide reasonable safety in descent on icy pavements. If commercial traffic is present in sufficiently large volume ruling grades may need to be limited in proportion to the economic importance of this class of traffic. It may be enlightening to study the operating efficiency and speed of various classes of trucks, loaded in accordance with current commercial practice, upon different rates and lengths of grades and allowing for motors not operating at "brand new" efficiency. It is well known that practically all trucks overload beyond the rated capacity of the vehicle. At present, manufacturers present data covering speeds and operating efficiency for different rates and lengths of grades, based on rated capacity and new motors. A test

program simulating actual commercial conditions, in co-operation with motor manufacturers, may change present conceptions somewhat as far as economic grade design is concerned, as applied to trucks.

Sight Distance.—The proper approach to this problem, if the dual highway principle is utilized, would appear to be related to the distance in which a vehicle can be brought to a stop. This is dependent on the braking power and the interval of time required for the operator to start the application of the brakes after he has observed a danger. This latter interval may be termed the lag. The result of computations based on the laws of motion and using the 1931 rules of the New Jersey Vehicle Commission for stopping distances indicates a deceleration factor of 17.4 ft per sec per sec for four-wheel brakes. This is slightly higher than the factor deduced (approximately 16 ft per sec per sec) from the figures of the National Bureau of Standards[2]. The formula, based on well-known principles of motion, is:

$$s = \frac{1.075\ V^2}{17.4} \quad \text{.........................(2)}$$

Equation (2) will give the distance, s, required to bring a vehicle equipped with four-wheel brakes to a stop, neglecting the lag. The lag has been observed by various authorities as ranging from 0.75 sec to 1.5 sec. for an emergency stop. In connection with the deceleration factor of 17.4, tests conducted by a New York, N. Y., Taxicab Company indicate that a deceleration of about 19.0 ft per sec per sec will throw the occupants of a car out of the seats. This suggests a limit to a further increase in braking power.

Applying Equation (2), using V equals 100 miles per hr, a distance of 617 ft will be required to bring a car to a stop (on level grade) after the brakes are applied. Using a lag of 1.5 sec, a sight distance of 836 ft is required (617 + 219), at a speed of 100 miles per hr. The sight distances calculated, using the factor, $d = 17.4$, should be considered the minimum.

Fig. 4 shows the stopping distances for different speeds and rates of grades. Let S = grade, in feet per foot ($-$ = down grade and $+$ = up grade); m = mass; and, d = the deceleration of the car, in feet per second per second; then, since total work = kinetic energy + potential energy:

$$m\ d\ s = \frac{m}{2}\left(\frac{5\,280}{3\,600}\ V\right)^2 + 32.2\ m\ (1 - S)\ s \quad \text{..........(3)}$$

For properly adjusted four-wheel brakes, using the New Jersey State traffic rules for 1931 a value of d equal to 17.4 is deduced from which it is possible to derive an expression for s in terms of S, as follows:

$$s = \frac{1.075\ V^2}{d + 32.2\ S} \quad \text{......................(4)}$$

If the dual highway principle is not used and the design consists of a two-lane, two-way highway, passing distance becomes the controlling element. For this case a sight distance of more than 2 000 ft may be required for

[2] Manual on Uniform Control Devices for Street and Highways, National Bureau of Standards, Appendix *C*, dated March 4, 1935.

reasonably safe passing. Even this distance does not take into consideration the speed of an approaching car. The passing menace is one of the serious problems facing the highway designer.

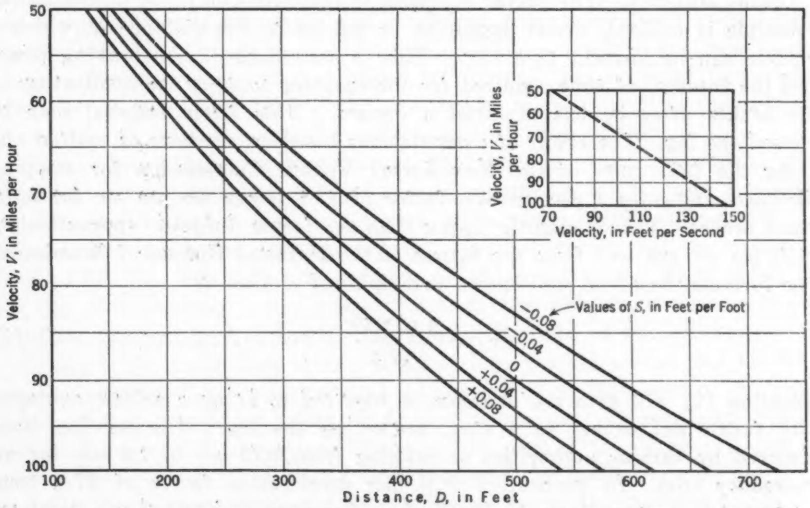


FIG. 4.—DISTANCE REQUIRED TO BRING A MOTOR CAR EQUIPPED WITH 4-WHEEL BRAKES TO A STOP AFTER BRAKES ARE APPLIED.

Shoulders.—If the shoulder is looked upon as a factor of safety a healthier attitude toward its cost will develop. In case of accident or mishap it may prove as important as the pavement itself. Many minor accidents develop into major accidents when cars swerve on to the shoulder and strike some fixed object, or over-turn. In many cases it is necessary for a car to dodge out on to the shoulder to miss a sudden accident ahead. For these reasons it is apparent the shoulders should be wide (preferably not less than 20 ft), level with the pavement, paved suitably to accommodate vehicles safely at high speed, and cleared of all obstructions. Ditches are recognized hazards and are being eliminated in present modern designs, but obstructions, such as poles, trees, warning signs, head-walls, light standards, etc., are not so generally recognized as sources of danger. The writer has often wondered why public officials permit private utility companies to locate poles within a few feet of pavements instead of along the right-of-way lines.

Curbing is another element of danger which has not been generally recognized. It is recommended that it be used very sparingly and only in places where it will prove to be a safety device instead of a hazard. It is well to keep in mind that the essential function of a curb is to protect pedestrians on sidewalks, to form drainage channels in urban areas, and as a first line of warning and defense on bridges, etc.

Signs.—The setting of warning signs as fixed objects near the roadway was probably derived from railway practice. Since the motor vehicle is not confined to fixed rails, signs set in this manner constitute a hazard.

Signs may either be suspended from overhead cables or mounted on flexible standards. To be effective they should be illuminated at night in such manner that the motorist is not blinded and so that objectionable shadows are not cast. The ideal would appear to be a luminous type visible for a sufficient distance, but with a limited light source. Reflectors are not always effective under certain conditions and, of necessity, must be set in a position to intercept head-light beams. In such a position the reflector usually becomes a menace.

A criticism frequently heard is that direction signs are not placed sufficiently in advance to permit the motorist time to make a turn properly and safely. This safe distance will be increasingly important because of advancing speeds. Braking power and feet traveled per second are the factors. Duplication of direction signs will also prove helpful.

Guard-Rail.—Probably no other single highway structure has been the cause of so much loss of life and property as the guard-rail. Because it is a hazard at best it is desirable that as little of it be placed as possible. For fills of moderate height it would appear logical, therefore, to substitute extra wide shoulders and flat side slopes. This would eliminate many miles of railing. In locations where the guard-rail is essential, only rail of such design that damage to the vehicle and injury to the occupant are avoided, should be installed.

The enlightening paper by Searcy B. Slack, M. Am. Soc. C. E.,[3] covering fundamentals of guard-rail design, points the way toward more rational design.

Lighting.—Practically all authorities agree on the present inadequacy and unsuitability of highway illumination. With increasing speeds and night travel this inadequacy assumes serious proportions. The lighting of to-day creates bright areas around which are pools of blackness difficult for the eye of the driver to penetrate. Indeed, some types of units defuse the rays laterally and upward leaving the pavement and an area some distance above the pavement in semi-shadow when compared with the brightness of the light itself. The effect on the motorist is to create a series of blind spots. Rain, snow, sleet, and fog increase this hazard. In this connection roadside stands with their brilliant lighting are very troublesome. Some of these stations produce a blind interval of several seconds. Proper legislation could eliminate this latter hazard. If highways are to be illuminated it is apparent that a radical departure from present methods will be necessary.

Pavements.—Present thought in pavement design appears concentrated on providing only sufficient strength, with a small factor of safety, to prevent actual failure. It may be that too little thought is being given to providing sufficient thickness and steel reinforcement to perpetuate the good riding qualities originally built into the pavement. The natural forces of weather, temperature, frost, and plastic flow in concrete tend to distort the riding surface. It is established that profileometer readings increase each successive year. Thus, after five or six years, the riding qualities are greatly impaired. The public is as interested in smoothness during the entire life of the pave-

[3] *Engineering News-Record*, June 9, 1932.

ment as in the first two or three years of its life. At present speeds, such roughness may not be unsafe, but at the speeds which may be common in the near future, it may be vital. A thick slab heavily reinforced top and bottom, with transverse joints less than 100 ft apart, may be the answer. Such a slab will be relatively stiffer and will have less deflection at transverse joints, and thus tend to lessen present troubles at the joints. A test program would do much to throw light on the problem.

At present, joints are receiving much deserved attention. They will be increasingly important in the future. Noise caused by the joint may be equally as important as smoothness. At a speed of 100 miles per hr with joints on 50-ft centers, a vehicle will pass over three transverse joints er second. If the joint causes noise it may prove more annoying and conducive to driver fatigue than the "clicking" of rail joints on the railways.

Terminals and Interchange Facilities in Urban Areas.—In such areas speeds may have to be limited to approximately 40 miles per hr in the interest of safety. Because of the specialized nature of the problem, the scope of this paper does not permit a detailed discussion. Each location presents individual design problems and must be treated on its merits. In the writer's opinion the simplest possible layout is best. A complicated design that confuses the motorist may be more conducive to lost time than a less mechanically perfect design that is simpler. The motoring public objects to "getting lost." The present tendency to eliminate all left turns on the secondary street may be unnecessary in most cases and may lead to undue complication. The decision as to the use of a design with no left turns depends on the peak-hour volume of traffic using the ramp and the street to be entered. If traffic on the street to be entered is light and not moving at high speeds, left turns may be permitted with safety. If the street to be entered is equipped with traffic lights the criterion will be that no more traffic shall accumulate in the ramp than will clear during a light interval. Observations show that at least ten cars in two lanes, will clear out easily during a 1-min. light interval. This is equal to not less than 300 cars per hr.

The principles of the traffic circle, the grade separation with ramps, and the clover leaf are well understood. The traffic-circle design is based on the volume of traffic as related to the diameter and the number of entrance streets, to the end that free weaving action may occur. The charge that traffic circles are dangerous is true if the circle is inserted in a high-speed highway where speeds are greater than 40 miles per hr. At speeds of about 40 miles per hr, a circle can be made safe with proper lighting, adequate illumined over-head warning signs placed sufficiently in advance, and by eliminating curbs and obstructions within and adjacent to the circle.

Center ramps, which require the main traffic to by-pass around them, are exceedingly dangerous. Heads of all turn-out ramps should be designed so that traffic does not tend to run into them. Curbs, if any, should be low and walls should be started sufficiently back from the nose to allow the motorist time to avoid crashing into them. Obstructions (signs, reflectors, light standards, etc.) placed on the nose are an added hazard. Proper lighting in the form of spot-lights and advance, over-head, illumined warning and direction signs

will be of assistance in making ramp heads safer. Many of them have been designed in improper positions, and then loaded with signs and reflectors. Such installations have resulted in many serious accidents with accompanying loss of life and property. No part of the design deserves a more careful accident analysis than ramp heads. It is not necessary for the motorist to enter the ramp at breakneck speed; nor is it necessary to use curves of large radius. The ramp should be a definite turnout, a subsidiary of the highway, and should be so designed. If properly "signed" far enough in advance, the motorist will find it even if it is off to one side of the highway.

It is well to repeat that simplicity of layout consistent with safety and traffic-handling capacity is sound policy when designing terminal facilities.

Conclusion

Most of the highways designed twelve or fifteen years ago (before about 1920) are obsolete as far as alignment, grades, and details are concerned. These shortcomings are the cause of untold suffering and loss of life as well as an enormous economic waste in property loss and capital outlay. Unless there is a radical revision upward in design standards, history may repeat itself as far as the trunk system is concerned. Motor-car design and the public will dictate the speeds which will obtain. The engineer will have little voice in the matter.

There are evidences in various parts of the country of an awakening to the principles outlined in this paper, but nothing really concrete has been initiated on a national scale. Germany, on the other hand, actually has under construction the first unit of a system designed for speeds as great as 115 miles per hr.

The highway engineer is facing a tremendous responsibility. He must have courage and vision. He controls the destiny of billions of dollars and tens of thousands of lives. Safety for the motorist must be the watchword.

Acknowledgments

Acknowledgment is freely given to the Delaware State Highway Department, Travelers Insurance Company, and Mr. E. B. Shrope for assistance and courtesy in supplying data and illustrations used in this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

SELECTION OF MATERIALS FOR ROLLED-FILL EARTH DAMS

By Charles H. Lee,[1] M. Am. Soc. C. E.

Synopsis

The traditional manner of determining suitability of material for earth dams is to exercise judgment, based upon previous experience, aided by sense testimony, such as experience, "feel" to the hand, and possibly grittiness between the teeth. In recent years, with increase in the number and size of earth dams and the occurrence of unexpected failure, it became apparent that a more scientific method of selection was needed. The first efforts in this direction were for hydraulic-fill dams where knowledge of particle size and other physical properties was found essential as a guide to safe and permanent construction.

The recent improvements in mechanical equipment for handling earth have so reduced the cost of rolled-fill construction that the rolled-fill dam is now an economic competitor of the hydraulic-fill dam. Due to this fact and to the freedom of the rolled-fill dam from the element of treachery during construction, it is becoming a more popular type of dam. The rolled-fill dam has this disadvantage, however, that if failure occurs, it does so when the structure is in service and after it has had full opportunity to absorb water. Due to this characteristic and to the fact that practical earth-testing methods have now become more or less standardized in other lines of technical activity, such as highway construction, the present tendency in rolled-fill construction is toward the use of scientific earth-testing methods. This tendency extends both to the selection of material and to control during construction.

This paper is confined to a discussion of earth testing as an aid to the selection of mateiials. Pertinent fundamental principles of soil technology as developed from the latest research and practice are drawn upon as a basis for conclusions. These principles are supplemented by certain research find-

Note.—Presented at the meeting of the Irrigation Division, Los Angeles, Calif., July 4, 1935. Discussion on this paper will be closed in December, 1936, *Proceedings*.

[1] Cons. Hydr. Engr., San Francisco, Calif.

ings in concrete technology. It is shown that particle size has a direct relation to the more important properties of earth material affecting its suitability for rolled-fill dam construction. Mechanical analysis, combined with microscopic examination and, in some cases, a simple mineralogical test, is proposed as a basis for determining suitability. Before making a final selection these factors may be confirmed by tests for compaction and permeability.

Requirements for Suitability

The principal requirements for suitability of material for the impervious part of a rolled-fill earth dam are:

(1) Permanent stability against sloughing when saturated;

(2) Reasonable water-tightness against the maximum head of water;

(3) Workability in construction operations, involving spreading and compacting in thin layers upon the top of the embankment by the use of mechanical equipment;

(4) Insolubility of the mineral constituents composing the individual particles of the material; and

(5) Reasonable cost of handling, including excavation, transportation, spreading, and compacting.

If the dam includes a permeable down-stream section, freedom of drainage must be considered for this part.

Stability

The essential requirement for stablization of earth is high shearing strength resulting from a proper combination of cohesion and internal friction. This can be secured in a well-graded material through mechanical compaction to the point of maximum density by tamping, rolling, or the tramping of animals. Internal friction in an earth material is produced by the rubbing together of bulky granular particles held in contact by the forces of cohesion. The presence of coarse material, such as gravel and sand, are essential for high internal friction.

Cohesion depends upon the permanent adhesive strength due to molecular attraction of minute films of water and moist clay particles between granular particles. As an illustration, C. A. Hogentogler, Assoc. M. Am. Soc. C. E., states[2] that an almost liquid clay with cohesion amounting to about 130 lb per sq ft, when combined with a cohesionless sand will increase the supporting power of the latter from 270 lb per sq ft to 2 500 lb per sq ft. Water films in liquid or plastic soils usually have the same characteristics as water in bulk, but when the thickness of the films is reduced by drying or mechanical compaction of the soil to a semi-solid or solid state, the boiling point of film water rises, the freezing point lowers, and the surface tension increases so that the films become tough. Charles Terzaghi, M. Am. Soc. C. E., states that

[2] *Public Roads*, February, 1935.

in thicknesses of less than 0.000002 in., water films behave like semi-solid substances[3].

The thickness of films on river sand passing a 10-mesh sieve has been estimated at 0.00002 in., and that passing a 60-mesh sieve at 0.000005 in.[4] This thickness decreases with the size of the particle, and in the case of clay is much less than for sand. As a result of this toughening of water films by the compaction of a plastic soil, the latter changes to a semi-solid or solid condition and acquires stability. If the size of the earth particles is sufficiently varied and a sufficiently high degree of compaction is obtained, this stability becomes permanent even when the material is submerged in water.

Active research in earth stabilization has been conducted during the past two decades, first in connection with highway construction, and more recently for rolled-fill dams. Research in the latter field has developed the fundamental principles of earth compaction[5], while the former among many other things has contributed a basis for the selection of materials for earth road surfaces[2]

Probably the most severe test for the stability of earth material is as a wearing surface for an earth road. Such a surface must remain unyielding and hard under pressure and the impact of the heaviest traffic; and it must be free from mud in wet weather or excessive dust in dry weather. Although the forces acting upon a dam are not concentrated as are those upon an earth road, they are of similar character. They consist principally of pressure from the weight of superimposed water and impact from waves and earthquake shock. An earth dam must also retain its stability under alternate submergence and atmospheric exposure. Because of the similarity in requirements for permanent stability in earth roads and in earth dams, the interchange of research data is admissible.

Without going into unnecessary detail, the conclusion may be stated that as the result of research, now thoroughly tested in practice, permanent stability in rolled-fill earth dams can be obtained with well-graded material compacted at a critical moisture content by use of appropriate mechanical equipment. The importance of this finding merits the following detailed consideration of both grading and compaction.

Grading.—A graded earth material is a mixture of earth particles regularly proportioned by size. The most desirable range of sizes and degree of grading differ with the type of construction and the service. For concrete aggregate a limited number of coarse particle sizes is used. For stabilized earth a wide variety of sizes, including both very coarse and very fine, is desirable. In earth-stabilization practice it is seldom possible from the economic standpoint to prepare artificially graded materials, so that natural materials must be used which conform as nearly as possible to an ideal grading.

The purpose of grading in earth stabilization is twofold: (1) To provide the mechanical properties of strength and hardness combined with cohesion and high internal friction; and (2) to secure maximum density.

[3] *Physical Review*, Vol. 16, 1920, p. 56.
[4] *Journal*, Am. Chemical Soc., Vol. 41, 1919, p. 477.
[5] *Engineering News-Record*, August 31, September 7, 21, and 28, 1935.

The mechanical properties of a well-graded earth result from diversity in particle size. The coarser particles held on a No. 10 sieve, including gravel, pebbles, and cobbles, may be considered as the coarse aggregate and the finer particles passing the No. 10 sieve and consisting of fine gravel, sand, silt, and clay, combined with water, may be considered as the mortar. The func'ion of the coarse aggregate and coarse sand is to provide structural strength and hardness and a high degree of internal friction. The fine sand provides a bedding for the coarse sand. Silt is a filler which prevents the granular particles from rocking. Clay occupies the remaining voids and reduces void spaces to such small dimensions that tough connecting water films are formed which produce strong cohesion. It is thus evident that fractions of all sizes from pebbles to clay are necessary to provide a full degree of strength, hardness, cohesion, and internal friction.

High density in a graded granular material results from the voids between larger particles being occupied by smaller particles. This agrees with the fact, well known to hydraulic engineers, that the granular materials of greatest porosity are those having uniform particle size such as filter sand or clay, whereas those of least porosity have a great variety of particle size. The effect of grading upon density is well illustrated by data published[6]

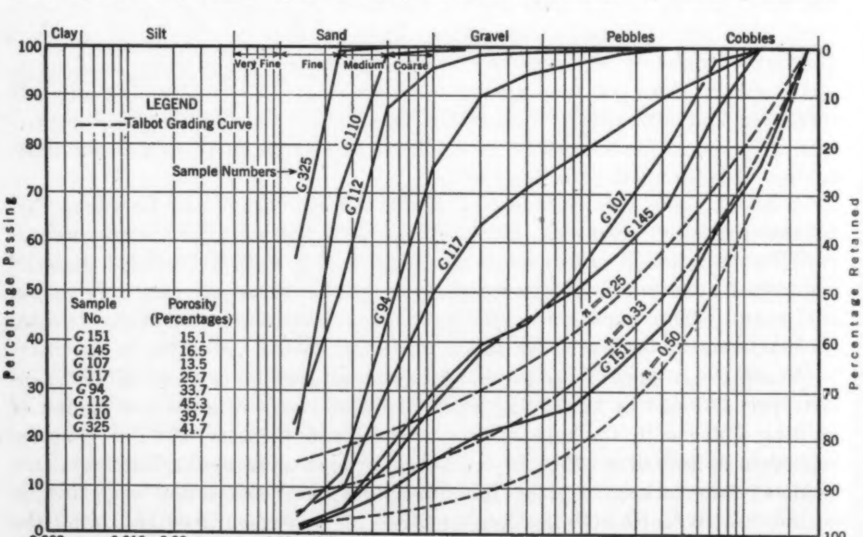

FIG. 1.—RELATION OF POROSITY TO GRADING.

in 1934 in a report giving mechanical analyses and porosities for several hundred undisturbed samples of alluvial material from the South Coastal Basin of Southern California. Data for eight typical samples from this report have been plotted in Fig. 1. Samples G–151, G–145, and G–107 con-

[6] *Bulletin No. 45*, Div. of Water Resources, State of California, South Coastal Basin Investigation, "Geology and Ground-Water Storage Capacity of Valley-Fill", 1934.

tain all sizes from large cobbles to fine sand, with a preponderance of coarse sizes and regular gradation of fine particles. The porosity of these three samples averages 15 per cent. Samples *G*–112, *G*–110, and *G*–325 are composed of fine and medium sand. These samples have porosities averaging 42%, and those with the intermediate ranges of sizes represented by Samples *G*–117 and *G*–94 have porosities between these two extremes.

Curves plotted to a semi-logarithmic scale which flatten out toward the fines and are concave upward represent well-graded material, whereas those which are flat in coarse material and become steep toward the fines, or are nearly straight and vertical, represent uniformity in particle size. The progressive decrease of porosity with grading is strikingly shown in Fig. 1.

The ideal grading for maximum density has been thoroughly investigated in the well-known experiments on aggregates and concrete by Sanford E. Thompson, M. Am. Soc. C. E., and the late William B. Fuller, M. Am. Soc. C. E., in 1903[7], supplemented by those of A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., and F. E. Richart, M. Am. Soc. C. E., in 1919 to 1923[8]. These experiments show that regular grading produced greater density than irregular grading, and that maximum density resulted from a relatively coarse grading As a result of his experiments, Professor Talbot developed a general grading equation which may be written, as,

$$p = \left(\frac{d}{D}\right)^{n} \qquad (1)$$

in which p = the proportion by weight passing a given screen or sieve opening; d = size of opening; D = maximum particle size; and n = an exponent. These experiments were conducted on graded mixtures with several maximum sizes of aggregates and values of n varying from 0.24 to 1.20, representing mixtures varying from extremely fine gradings to extremely coarse. Characteristics of gradings for typical values of n, as shown by these experiments, are as follows:

For $n = 1.0$, the curve of Equation (1) is a straight line when plotted to natural scale and represents a very coarse grading;

For $n = 0.5$, the curve is a parabola (natural scale) and represents a medium grading corresponding with the Fuller-Thompson parabolic grading (provided sieve holes are of the same shape); it produces maximum density of mixture for aggregates having a maximum size of approximately $\frac{1}{4}$ to $\frac{1}{2}$ in.

For $n = 0.33$, the curve is more nearly an ellipse (natural scale) and represents a fine grading with maximum density of mixture (porosity, 20 to 22%) for aggregates having a maximum size of from 1 in. to 2 in.; and,

For $n = 0.25$, the curve also resembles an ellipse and represents a very fine grading with maximum density of mixture for aggregates having a maximum size of from 4 to 6 in.

The Talbot grading equation, although developed for use with concrete aggregates, is applicable to earth mixtures and may be used as a criterion

[7] *Transactions*, Am. Soc. C. E., Vol. LIX (1907), p. 67.

[8] *Bulletin No. 137*, Eng. Experiment Station, Univ. of Illinois, October 15, 1923.

for density in the selection of natural earth material for rolled-fill dams. This is shown by the following two examples.

Example 1.—Grading curves with 10-in. (256-mm) maximum size of particle, when plotted in Fig. 1 correspond in shape with the mechanical analysis curve for Sample *G*–151, the curve for $n = 0.33$ corresponding most closely in position. A similar plotting with a 5-in. (128-mm) maximum size shows that the grading curve for $n = 0.33$ also corresponds generally in shape and position with the mechanical analysis curves of Samples *G*–145 and *G*–107. These three samples all have high density (average porosity, 15%). Similar plotting against the mechanical analysis curves of five other samples, all of which have lower densities, shows no similarity in shape or position. In other words, the Talbot grading equation automatically designates the material of highest density.

Example 2.—Based upon extensive experience, the United States Bureau of Public Roads recommends the following size specification for mixtures having sufficient stability for earth road surfacing[9]:

Sieve.	Percentage passing by weight.	Sieve.	Percentage passing by weight.
1-in.	100	No. 10	40 to 65
$\frac{3}{4}$-in.	85 to 100	No. 40	25 to 50
No. 4	55 to 85	No. 270	10 to 25

The soil mortar in this material passing the No. 10 sieve has a composition as shown in Table 1[10]. These limits have been plotted in Fig. 2, together

TABLE 1.

Fraction	Diameter, in millimeters	Percentage by weight
Clay	Less than 0.005	5–10
Silt	0.005 to 0.05	10–20
Sand, total	0.05 to 1.0	70–85
Sand, coarse	0.5 to 1.0	45–60
Average effective size	0.01	.....
Uniformity coefficient	Greater than 15	.....

with grading curves for the corresponding maximum particles sizes. Here, again, there is found to be a very close agreement in shape and position, the curve for $n = 0.33$ lying closest to the mechanical analysis curve for the mixture, and $n = 0.50$ closest to the corresponding curve for the soil mortar. This shows that the Talbot grading curves for maximum density designate an earth mixture found to be satisfactory for the surfacing of earth roads.

From these and other studies it is concluded that the Talbot grading curve with values of n between 0.25 and 0.50 may be used as a criterion for high density in the selection of natural material for rolled-fill dams.

[9] *Public Roads*, February, 1935, p. 277.
[10] *Loc. cit.*, July, 1931.

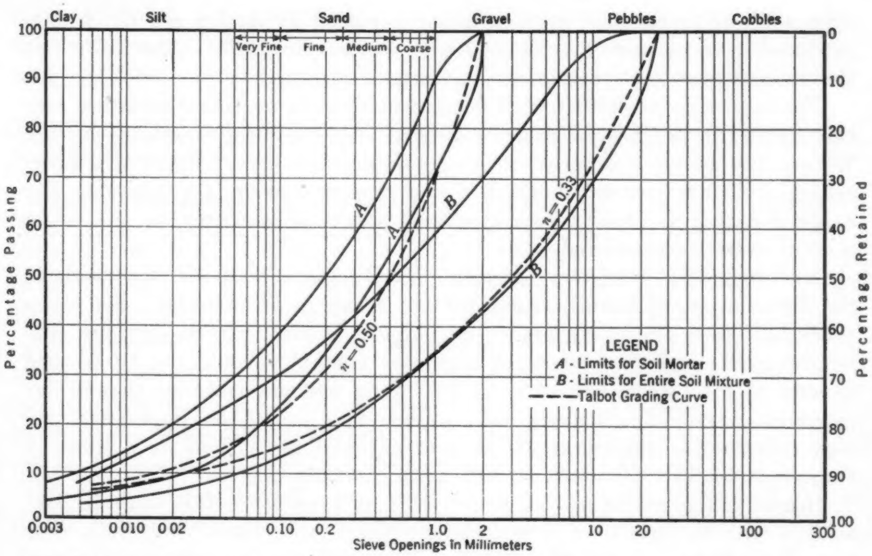

FIG. 2.—SIZE SPECIFICATION OF MIXTURES RECOMMENDED BY U. S. BUREAU OF PUBLIC ROADS FOR SURFACING EARTH ROADS.

COMPACTION

The purpose of compaction of earth material is to bring the soil particles into uniform close contact. This has three effects: First, it increases the mechanical bond by interlocking the individual grains; second (and, of much more importance), it decreases the thickness of connecting water films, thus greatly increasing their adhesive force; and third, it partly expels air which, if retained, would ultimately be replaced by water, thus increasing the thickness of water films and decreasing cohesion. The net effect of compaction is to increase density and produce permanent cohesion.

In open areas earth is compacted by spreading it in thin layers and rolling it with trucks and heavy flat rollers, or kneading and rolling it with sheepsfoot rollers. In restricted spaces the material is made compact by tamping.

The fundamental principles of soil compaction and their application to rolled-fill earth dams in the selection of materials and control during construction have been thoroughly developed by R. R. Proctor, M. Am. Soc. C. E.[11] His work shows that in a given earth material and for each particular method of compaction, there is a critical moisture content at which maximum density is attained. Higher water content results in increased plasticity, larger voids, and less density, until a point is reached at which equipment will not be supported. A lower content may produce a fill of greater apparent stability with little or no plasticity, but one having a greater void volume and less density. The decreased water content and increased void volume in

[11] *Engineering News-Record*, August 31, September 7, 21, and 28, 1933.

such a fill is represented by air. After a period of service the air will be displaced by water and the material will become saturated. This will soften the fill and may finally produce sloughing or piping.

For each type of construction equipment there is a critical moisture content which will produce the proper degree of lubrication for maximum density. With a heavier or a different type of equipment, however, there is a lower critical moisture content at which a still greater density is attainable. For a given degree of density both type and weight of construction and water content must be considered.

Although they were very complete, Mr. Proctor's tests did not result in the formulation of definite rules for the selection of material. He states that "soils that are poorly graded and almost lacking in fine particles cannot be used"; and that "those of high clay content" should not be selected because of the higher cost of handling. It is obvious, however, that a clay material of very fine texture such as bentonite would be impractical for dam construction regardless of the cost of handling. There is a limit in fineness of texture, therefore, as well as in coarseness.

Recently, the writer had an opportunity to investigate this limit in connection with the selection of material for the Coyote Dam being built near San Jose, Calif., by the Santa Clara Valley Water Conservation District. Among the materials selected for testing was one of ancient lacustrine origin. With the passage of time this material had become indurated to a siltstone, was subsequently elevated, and, finally, became softened by weathering, to a depth of more than 12 ft. It was located conveniently for handling and in preliminary field examination had been favorably considered for use in rolled-fill construction of the impervious section of the dam. When samples from the weathered zone were subjected to mechanical analysis by the sieve and hydrometer method, difficulty was experienced in breaking the material down to ultimate particle size during the ordinary period of stirring, even with pre-treatment. Microscopic examination of particles after stirring, as held by Tyler standard sieves from No. 48 to No. 270, showed numerous rounded aggregations of silt and clay representing particles of original siltstone. These particles were soft and more or less plastic, and after drying crushed easily under the thumb-nail. With long-continued further stirring these aggregates finally broke down into particles of silt and clay size. It was also found that larger unbroken pieces of the original material absorbed water rapidly and softened so as to crush easily between the fingers.

The question then arose as to whether this material would act in the completed fill as a mixed sand, silt, and clay, which was the constituency as indicated by field examination and standard mechanical analysis, or whether it would act in accordance with ultimate particle sizes as obtained by prolonged stirring. The latter indicated a material of finer texture than that of the borrow-pit samples from Alexander Dam, in Hawaii, and closely resembled the core samples from that dam. The experience with such fine material for rolled-fill construction both in the Islands and elsewhere has been very unsatisfactory[12]. This is illustrated by the clause often placed

[12] *Engineering News-Record*, October 20, 1932, p. 468.

in specifications for rolled-fill dams, requiring exclusion of adobe soils. It was determined, therefore, to make a series of compaction tests, including materials from very fine to very coarse texture, and to observe especially the

TABLE 2.—Description of Samples Tested at Coyote Dam, in California

Sample	Classification	Type of material	Location	Project	True specific gravity
*L*11–1	Clay	Silt	Bed of reservoir	Zuni Reservoir, N. Mex.	2.89
*L*8–*A*	Clay	Antioch clay loam	Borrow-pit	Port Costa Dam, Calif.	2.88
*L*15–*CG*	Silty clay	Siltstone	Sample pit	Coyote Dam, Calif.	2.88
*L*15–*A*1	Sandy clay	Weathered sandstone	Sample pit	Almaden Dam, Calif.	2.75
*L*15–*C*3	Sandy clay	Weathered sandstone	Sample pit	Calero Dam, Calif.	2.75
*L*15–*C*9	Clay loam	Weathered Franciscan sandstone	Sample pit	Coyote Dam, Calif.	2.72
*L*15–*S*9	Fine sandy clay	Alluvial sand	Sample pit	Stevens Dam, Calif.	2.71
*L*15–*S*6	Coarse sandy loam	Disintegrated Santa Clara formation	Sample pit	Stevens Dam, Calif.	2.76
*L*15–*A*5	Coarse sandy loam	Weathered sandstone	Sample pit	Almaden Dam, Calif.	2.73
*L*15–*S*10	Sandy gravel	Weathered Santa Clara formation	Sample pit	Stevens Dam, Calif.	2.89
*L*1–1	Fine sand	Beach sand	Cliff House Beach, San Francisco, Calif.	....	2.93

extent to which air remained in the compacted samples. Ten samples were selected for testing as described in Table 2. Mechanical analyses of samples were made by the standard hydrometer and sieve method as developed by the

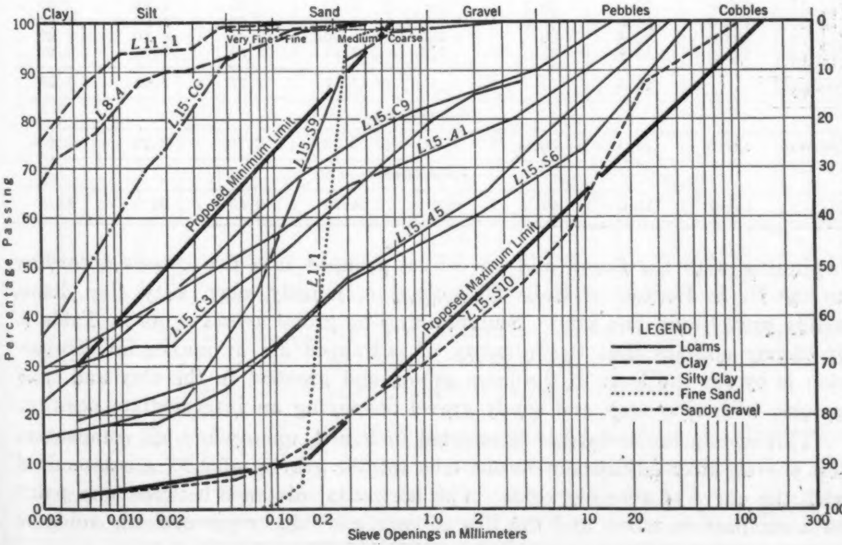

Fig. 3.—Mechanical Analysis of Samples Tested in Compaction.

U. S. Bureau of Public Roads[13]. In the case of Sample *L*15-Coyote *G*, composed largely of siltstone, the suspension was shaken for 17 hr on a revolving wheel and correction was made after a microscopic examination and a count of undigested siltstone particles in the various screen fractions. The results of mechanical analyses are shown graphically in Fig. 3.

Compaction tests were made on all samples in conformity with the standard laboratory method developed by Mr. Proctor, the results being listed in Table 3. The samples have been arranged in this table in order of texture,

TABLE 3.—SUMMARY OF COMPACTION TESTS

Sample	At Maximum Compaction				Average Air Content, with Water Content Greater than at Maximum Compaction		Air Content at Maximum Compaction	
	Dry weight, in pounds per cubic foot	Water content, percentage by weight	Porosity, percentage by volume	Needle penetration resistance, in pounds per square inch	Percentage by volume	Equivalent water, percentage by weight	Percentage by volume	Equivalent water, percentage by weight
				CLAY				
*L*11-1..	100.5	20.8	44.5	1 490	6.03	4.21	10.9	6.80
*L*8-*A*...	88.7	27.0	51.1	1 520	8.06	6.40	12.5	8.80
Average	.....	23.9	....	.....	7.04	5.30	11.7	7.80
				SILTY CLAY				
*L*15*C*-*G*	103.0	21.5	43.0	1 030	5.14	3.40	7.58	4.60
				LOAM				
*L*15-*A*1.	116.0	15.7	32.9	750	3.25	1.86	3.25	1.86
*L*15-*C*3.	114.0	16.6	33.0	750	2.70	1.54	3.10	1.70
*L*15-*C*9.	120.0	13.8	29.6	840	2.50	1.35	2.86	1.40
*L*15-*S*9.	118.0	14.0	31.0	1 200	4.43	2.46	4.40	2.40
*L*15-*S*6.	123.0	12.3	29.3	1 460	3.90	2.10	4.70	2.20
*L*15-*A*5.	114.0	16.7	33.4	770	2.23	1.33	3.20	1.60
Average	.....	14.8	....	.....	3.17	1.77	3.58	1.86
				SANDY GRAVEL				
*L*15-*S*10	127.0	11.1	30.0	2 300	5.13	2.70	7.35	3.60
				FINE SAND				
*L*1-1...	111.0	11.0	40.2	570	14.50	8.45	20.3	12.3

beginning with the finest, and have been grouped into five classes according to the U. S. Bureau of Soils Classification, namely, clay, silty clay, loam, sandy gravel, and fine sand. Summarizing by group averages, as in Table 4, it clearly appears that the quantity of entrapped air at maximum compaction is by far the least in the loam group, and greatest in the clay and sand groups, with silty clay and sandy gravel occupying an intermediate position.

This conclusion is further illustrated in Fig. 4, upon which all compaction test curves, after adjustment to one true specific gravity of 2.80, are assembled with the curve of zero air voids. The horizontal distance between any point on a compaction curve and the line of zero air voids represents air voids as

[13] *Public Roads*, October, 1931.

TABLE 4.—COMPACTION TESTS, COYOTE DAM, IN CALIFORNIA

Class of material	(a) PERCENTAGE OF TOTAL VOIDS AT MAXIMUM COMPACTION OCCUPIED BY AIR					(b) PERCENTAGE INCREASE IN WATER CONTENT AT MAXIMUM COMPACTION IF AIR IS DISPLACED BY WATER	
	Limits in proportion of clay, percentage	Water content, in percentage by weight	Porosity, in percentage by volume	Air content, in percentage by volume	Percentage of total voids represented by air	Air content; equivalent water as percentage by weight	Increase in water content if saturated; percentage by weight
Clay	30 to 100	23.9	47.8	11.7	24.5	7.80	32.6
Silty clay	30 to 50	21.5	43.0	7.6	17.6	4.60	21.4
Clay loam	20 to 30	14.8	31.5	3.6	11.4	1.86	12.6
Sandy gravel	15 to 0*	11.1	30.0	7.4	24.6	3.60	32.4
Fine sand	15 to 0*	11.0	40.2	20.3	50.5	12.30	112.0

* Silt and clay.

equivalent water content, in percentages of dry weight. Each texture group has a distinctive type of compaction curve and occupies a distinctive area upon the diagram, thus indicating differing physical properties. Compaction

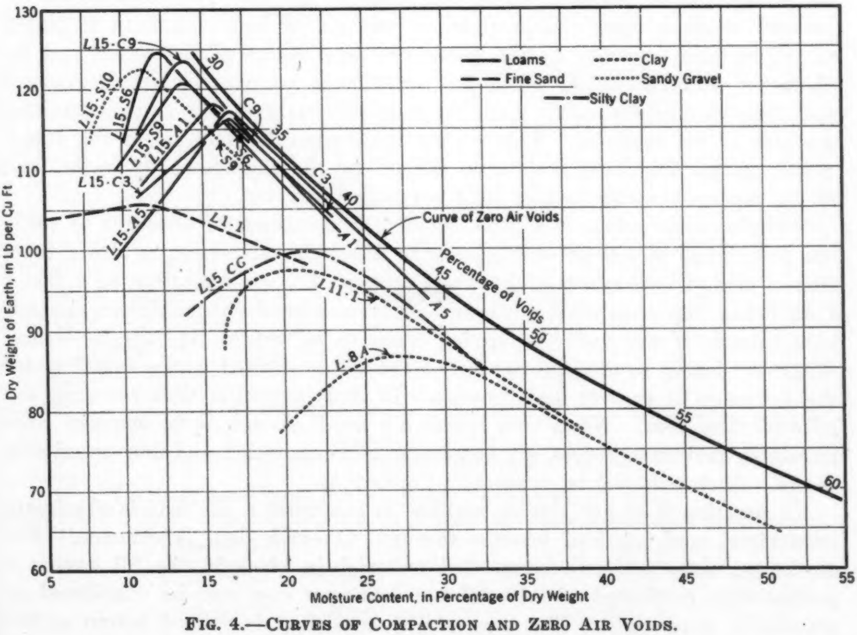

FIG. 4.—CURVES OF COMPACTION AND ZERO AIR VOIDS.

curves for clay, silty clay, and sand groups, are farthest from the line of zero air voids and lowest in dry weight at maximum compaction; and they have very flat peaks. The loam group is the closest to the line of zero air voids, has the greatest dry weight, and its curves have very sharp peaks. The sandy gravel curve, although having a sharp peak, and dry weight similar to those

for loam, is farther from the curve of zero air voids. The superiority of loam from the standpoint of dry weight and minimum air voids is apparent.

Expressing this superiority in terms of mechanical analysis, the data in Figs. 3 and 4 indicate that for permanent stability of compacted earth material the clay fraction should not exceed 30 to 35%; nor should it be less than 3 to 15%, the latter limits depending on the degree to which the material is graded.

With regard to the disintegrated siltstone (Sample *L*15-*CG*), it appeared that the air content at maximum compaction expressed as equivalent water was three times that of the next nearest available material represented by Sample *L*15-*C*9 (Fig. 4) and that it lay in the clay group in Fig. 3. Due to the large percentage of air voids, even under ideal conditions of handling in the laboratory, and the probability of non-uniform compaction in the fill due to difficulty in handling, the siltstone was discarded for use in the main structure of the dam.

IMPERMEABILITY

The water-tightness of an earth dam is determined by the degree of impermeability of the material as compacted in the fill. This impermeability, in turn, depends upon the gradient, or relation of hydraulic head to length of soil column traversed by the water, the temperature of the water, and the character of the material. Since the variation in range of maximum gradient and temperature for earth dams is relatively narrow, the most important variable is the material. This affects impermeability both from the standpoint of the frictional resistance offered to the movement of water, and of the molecular attraction of solid particles for water.

Frictional resistance is a function of the smoothness of the walls of voids, but principally of size of void openings. Molecular attraction is also a function of size of void spaces, although it does not become important as a factor influencing impermeability until void spaces are small enough for an appreciable volume of the water occupying them to be within its effective range. When the latter occurs, the portion of the water affected ceases to act under the influence of gravity and becomes stiff and immobile, thus reducing the effective flow area. When void spaces are small enough to be spanned completely by molecular forces, all water acts as a semi-solid, and flow practically ceases. Such material is completely impervious.

In practice it is not always possible to construct a fill that is absolutely impervious, and nominal leakage through an earth dam is common. For earth-dam construction a compacted material in place in the fill having a permeability coefficient of 0.10 gal per sq ft per day may be considered as practically water-tight. The permeability coefficient is defined herein as the quantity of water, in gallons per day, flowing through a section of material perpendicular to the direction of flow and 1 sq ft in area under a hydraulic gradient of 100% and temperature of 60° F. A permeability coefficient of 0.1 as thus defined is equivalent to a percolation rate, as defined by Proctor, of 4.86 ft per yr. For an earth dam of uniform construction 200 ft in height, and 650 ft in average width, with a slope of 1 on 2.5, a top width of 20 ft, and

a 10-ft free-board (assuming the water-table just below the surface of the ground at the down-stream toe), a permeability coefficient of 0.1 gal per sq ft per day represents a leakage, when full, of approximately 2 600 gal per day, or 0.004 cu ft per sec. Under certain conditions permeability coefficients exceeding 1.0 gal per sq ft per day might be allowable.

The recent standardizing of methods for measuring and expressing permeability make possible a correlation of mechanical analysis and permeability coefficients. Based upon the study of numerous permeability tests made by the Pacific Hydrologic Laboratory, and Hydrologic Laboratory, Water Resources Branch, United States Geological Survey, the statement can be made that regularly graded material having a degree of fineness sufficient to include at least 3% of clay, or ungraded material containing at least 25% of clay, when thoroughly compacted, is practically water-tight. Partly or poorly graded materials may be impervious if they contain less than 25% and

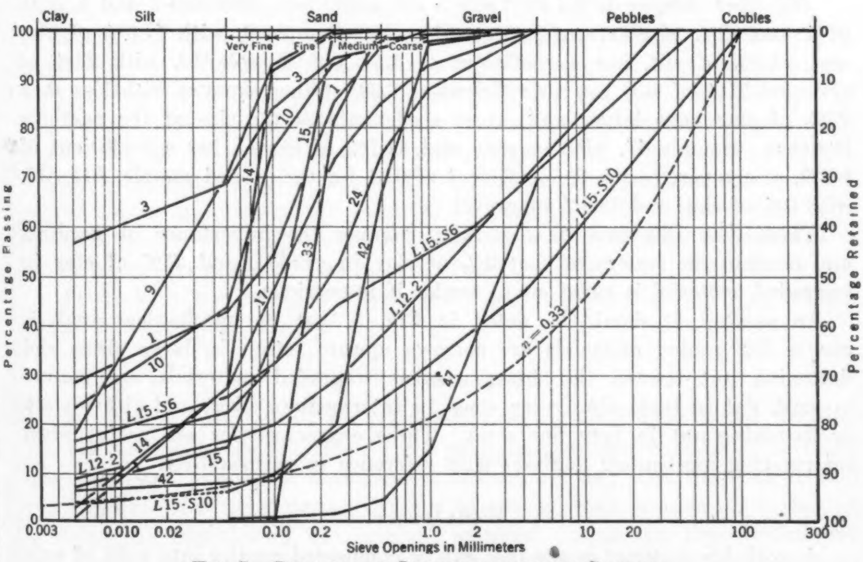

FIG. 5.—RELATION OF IMPERMEABILITY TO GRADING.

TABLE 5.—PERMEABILITY DATA PERTAINING TO FIG. 5

Sample	Permeability coefficient, in gallons per square foot per 24 hours (100% gradient)	Porosity (percentage of voids)	Sample	Permeability coefficient, in gallons per square foot per 24 hours (100% gradient)	Porosity (percentage of voids)
L15-S6*	0.034	29.3	14	13	34.9
L15-S10*	0.010	30.0	15	13	44.5
L12-2*	0.516	....	17	18	53.7
3	0	36.2	24	131	36.0
1	0	35.8	33	375	40.2
9	1.6	37.8	42	1 095	44.4
10	2.28	35.2	47	6 200	31.9

* The first three samples were graded approximately; the remainder were not graded.

more than 3% of clay, the quantity depending upon the departure from regular grading. These conclusions are illustrated by mechanical analysis and permeability data for a wide variety of samples as presented in Fig. 5 and Table 5.

Samples *L*15-*S*10 and *L*15-*S*6 closely approximate regular grading, the mechanical analysis curves being roughly parallel to the regular grading curve for $n = 0.33$ (see Fig. 5). Sample *L*12-2, which conforms more nearly to a straight line, also has a rough grading, although not to the same degree as the other two. Sample *L*15-*S*10, which approaches nearest to the ideal curve, contains only 6.5% of silt and clay and approximately 3% of clay. Its permeability coefficient is 0.01. Sample *L*15-*S*6, which is less well-graded, has a permeability coefficient of 0.034, and a clay content of 17. Sample *L*12-2, which is poorly graded and has only 9% of clay, has a permeability coefficient of 0.5.

All other samples listed in Table 5 are ungraded. Samples 3 and 1, with 57% and 29% of clay, are impermeable; but Sample 9, with 17.3% of clay and 53.1% of silt, has a coefficient of 1.6; and Sample 10, with 23% of clay and 21% of silt, has a coefficient of 2.28. Other samples with less than 25% of clay have larger and larger coefficients as the size of the particles increase. Sample 47, with no clay and 54.7% of gravel, has a coefficient of 6 200, as compared with the coefficient of 0.01 for the graded sample, *L*15-*S*10, with 3% of clay and 68.6% of gravel.

These data illustrate in a striking manner the importance of grading, and confirm the statement that 3% of clay in graded, and 25% of clay in ungraded, material is sufficient to render it impervious.

In passing, it should be noted in Fig. 5 that the mechanical analysis curves for graded materials are concave upward, steep in large sizes, and flattening out toward the fines; whereas ungraded materials are convex upward, flat in large sizes, very steep in intermediate sizes, and either steep or flattening out in very fine sizes. These characteristics are useful when interpreting mechanical analyses with reference to permeability.

WORKABILITY

A workable material is one that can be compacted readily into a fill of uniform texture and high density by the use of mechanical equipment. In order to fulfill this requirement the material must compact easily, and not be so harsh as to cause the equipment to wear out rapidly, not so sticky as to delay the spreading and rolling operations. These qualities are controlled largely by the texture of the material.

Materials that compact most easily are composed of well-graded gravel and sand with a silt and clay percentage of from 10 to 25 to supply lubrication. The application of proper moisture percentage for maximum density also facilitates compaction.

Harshness is produced by coarseness of grading either with or without the maximum size of particles, such as large pebbles and cobble-stones. The maximum allowable size of stone varies with the thickness of the layers

and the type of rolling equipment, whether flat or sheepsfoot. With 6-in. layers, cobbles should not exceed 5 in. in diameter. The allowable coarseness is also related to the type of equipment. As a general practice, values of n in Equation (1) should not exceed 0.50.

Stickiness is produced by high clay content. The allowable quantity can be varied, depending upon the type of equipment and time at which moisture is applied. In general, clayey material with a moisture percentage giving maximum density in compaction is not sticky in handling, provided the moisture is uniformly distributed through the mass. Clayey material is very slow to absorb water, however, and if worked immediately after sprinkling may be very sticky, even if the proper quantity of moisture has been added. This is due to the fact that the moisture content of the exposed surface with which equipment comes in contact is temporarily far greater than the critical moisture. If several hours are allowed after sprinkling for complete absorption and distribution of moisture, no difficulty may be experienced.

There are also other methods of overcoming stickiness which do not involve delay. The sprinkling of the rolled surface of the previous layer immediately in advance of dumping material for the new layer has proved successful. The exposed surface of the new material remains comparatively dry during the process of spreading and initial rolling, attaining an ultimate moisture content not in excess of the critical moisture after the water has moved slowly upward through the layer by capillarity. Another method is to sprinkle the material in the bank before excavation if natural moisture is much below the critical. In some instances, rainfall will moisten the soil in the bank or borrow-pit to a sufficient depth to bring the moisture content up to the percentage desired.

Insolubility

An obvious requirement for permanency in an earth dam is that the substances composing the individual earth particles must not be soluble. Gypsum and lime are frequently encountered in sedimentary rock formations and may appear in alluvial deposits either as incrustation or nodules occupying voids, or as particles of the original material. Although these substances are not as readily soluble as rock salt or alkali incrustations, percolating water will dissolve and remove appreciable quantities in time, rendering the fill more porous and threatening its stability and water-tightness.

The presence of soluble substances can usually be detected by simple observation, aided sometimes with a pocket microscope or an acid test.

Cost of Handling

Other requirements having been satisfied, the cost of handling the material is of final importance. Handling includes the operations of excavation, transportation, spreading, and rolling. Important factors affecting the cost of material taken from a given bank or borrow-pit are the distance and relative elevation from the dam, the quantity of satisfactory material avail-

able with reference to the volume of the dam, and the clay content of the material. The relation of these factors to cost is well understood and needs no discussion.

Mechanical Analysis as a Basis for Selection

Of the five requirements for judging the suitability of material for rolled-fill earth dam construction (cited previously), the three most important are stability, water-tightness, and workability. There is an intimate relation between each of them and the mechanical composition of the material. Stability and water-tightness are both secured as a result of regular grading of particle sizes and compaction of material; grading is entirely a matter of mechanical composition. In order to be effective permanently, compaction necessitates the establishment of limits for the clay and silt fractions. Workability also involves limits, both for fine and coarse fractions. This suggests the possibility for using mechanical analysis as a preliminary basis for the selection of material.

Examination of mechanical analysis curves for a variety of materials, such as those shown in Figs. 1 and 5, discloses great divergence in slopes, curvature, and terminal points. Even between fixed terminal points an almost infinite number of curves are possible. Such curves represent material varying from well-graded to uniform size of particle, from dense to very porous, and from impermeable to highly permeable. As these extremes result not only from variation in particle size, but also from variation in grading, it is apparent that if any method of selection based on mechanical analysis is to be successful, it must include limits of size as well as of departure from regular grading. If specified from the graphical standpoint, this means that in referring to the mechanical analysis diagrams, limits in shape and slope of curvature must be considered as well as limits in area. To establish such limits, it is proposed, first, to set up limits of particle size for coarse and for fine materials; and, second, to prescribe curvature limits based upon both degree and fineness of grading. The establishment of a size limit for coarse materials has already been attempted[14] by E. W. Lane, M. Am. Soc. C. E., but, to date, no attempt has been made to establish limits for size of fine materials, nor for degree and fineness of grading.

Before proceeding to details, it should be noted: First, that the terminal points of a mechanical analysis curve indicate size limits; second, that the slope of a mechanical analysis curve indicates the fineness of grading as numerically represented by the value of n in Equation (1); and, third, that the shape of a mechanical analysis curve indicates the degree of grading, if any, as numerically represented by departure from Equation (1). It thus appears that two sets of limiting curves are desirable, one applying to regularly graded materials and the other to ungraded materials. Between these limits will lie the poorly graded materials. Figs. 6 and 7, with Table 6, illustrate the range of the proposed specifications for limiting curves for graded and ungraded materials.

[14] *Engineering News-Record,* December 18, 1930, p. 961.

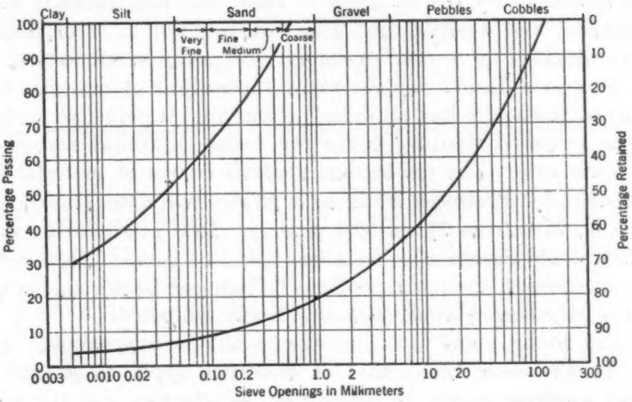

FIG. 6.—PROPOSED LIMITS IN MECHANICAL ANALYSIS FOR GRADED MATERIALS SUITABLE FOR IMPERVIOUS SECTIONS OF ROLLED-FILL EARTH DAMS

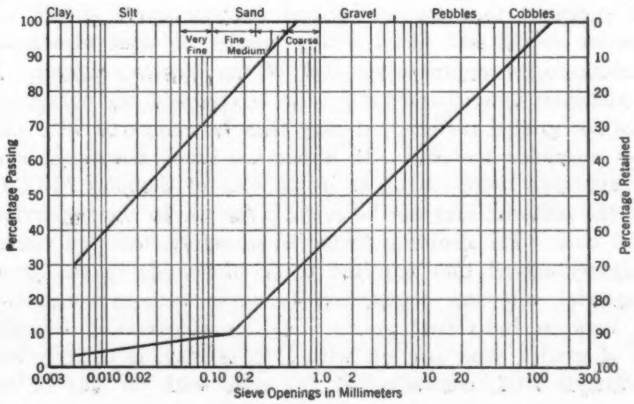

FIG. 7.—PROPOSED LIMITS FOR UNGRADED MATERIALS SUITABLE FOR IMPERVIOUS SECTIONS FOR ROLLED-FILL DAMS

TABLE 6.—PROPOSED LIMITS FOR GRADED AND UNGRADED MATERIALS

Class of material	Terminal Points for Curves (See Figs. 6 and 7)		Graded materials; values of n in Equation (1)	Ungraded materials; Remarks
	Maximum particle size, in millimeters	Minimum percentage of clay ‡		
Coarse	128*	3 to 5	0.33	Straight lines from terminal points to 10% less than 0.15 mm.
Intermediate	128* to 0.6†	3 to 30	0.33 to 0.25	Approximately parallel to the fine limit and the upper arm of the coarse limit, but not crossing the lower arm of the coarse limit.
Fine	0.6	30	0.25	Straight line.

* 5 in. † No. 28 mesh. ‡ Particle size, 0.005 mm.

These specifications are designed to fulfill the requirements for stability, water-tightness, and workability. They are believed to incorporate standard practice as modified by recent laboratory research in earth testing. Comparison with Professor Lane's "approximate upper limit of material suitable for impervious sections of rolled-fill dams", shows that it coincides with the upper arm of the proposed limiting curve for coarse ungraded material, but not with the lower arm. The mechanical analysis curves of material from existing dams used by Professor Lane as a guide terminate at 0.07 mm in the sand fraction, with from 6% to 10% passing. It is possible that these curves, if extended, would show a clay fraction of 3%, or more. If so, and this fraction were considered, the lower portion of Professor Lane's approximate upper limit might have corresponded with that herein proposed.

It is also to be noted in connection with the approximate upper limit proposed by Professor Lane, that it does not apply to graded materials. Mechanical analysis curves of the latter, satisfactory for the construction of rolled-fill dams, may lie beyond this limit in the sand, gravel, and pebble fractions, provided such material contains at least 3% of clay.

In applying the proposed limits to determine the suitability of a material, it is not sufficient that the mechanical analysis curve should lie between the respective coarse and fine limiting curves; it must also have a shape corresponding to, or approaching, that of the limiting curves. If it does not have sufficient upward concavity to fit or approach the curvature of limiting curves for graded material, it may then be compared with the limiting curves for ungraded material. If it has too much downward concavity to follow or approach these curves, as in the case of Samples *L*15-*S*9 and *L*1-1 in Fig. 3, the material may not be suitable for use in the impervious part of a rolled-fill dam. The deciding factor is, therefore, the clay content, which should slightly exceed that required to fill the voids in the granular part of the material, allowing compaction to occur without completely closing the gaps between individual grains. As an illustration, Sample *L*15-*S*9, composed of graded sand and silt with 17% of clay, is entirely satisfactory, whereas Sample *L*1-1, consisting of fine sand with no clay or silt, is not suitable.

The selection of material from mechanical analysis data, although believed to be practical, is more or less preliminary in character at present and subject to modification as the result of more positive tests, such as compaction, permeability, etc.

These criteria for selection of earth material for rolled-fill dam construction are presented in the hope that they will bring forth discussion and additional data, from which may be evolved a basis for preliminary selection generally acceptable to the profession.

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PAPERS

INTERACTION BETWEEN RIB AND SUPERSTRUCTURE IN CONCRETE ARCH BRIDGES

By Nathan M. Newmark,[1] Jun. Am. Soc. C. E.

Synopsis

Certain phases of the interaction between the rib and spandrel structure of concrete arch bridges are discussed herein with a view toward relating the action of the structure to the system of classification of indeterminate structures presented in 1935 by Hardy Cross[2], M. Am. Soc. C. E.

Certain limiting cases of the arch with integral superstructure are normal and can be designed directly from the results of a preliminary analysis. Structures in which either the rib or the deck girder develop all the flexural resistance are normal. A third type of structure having normal characteristics for most cases of loading is similar to a Vierendeel truss, with rigid columns and with floor and rib of approximately equal stiffness.

Other structures are hybrid and cannot be designed for given working stresses by the traditional procedure of repeated analysis and design. However, where the columns are flexible the action of the structure can be predetermined and the stresses can be predicted and controlled within reasonable limits.

In the case of the hybrid structure, the same degree of economy and safety can generally be obtained by designing the rib to carry all the load, and by neglecting the effect of interaction on the deck, as can be obtained by taking account of interaction by the traditional procedure of repeated analysis and design.

Introduction

The usual treatment of an open-spandrel arch bridge as a rib without restraint due to the superstructure is clearly incomplete. Interaction between

Note.—Discussion on this paper will be closed in **December, 1936,** *Proceedings.*

[1] Research Associate in Civ. Eng., Univ. of Illinois, Urbana, Ill.

[2] "The Relation of Analysis to Structural Design," Hardy Cross, *Proceedings,* Am. Soc. C. E., October, 1935, p. 1119.

rib and spandrel structure is inevitable as long as the parts of the structure remain continuous; but the fact that arch bridges designed by a procedure neglecting deck participation have apparently given good service, seems to indicate that interaction is relatively unimportant. However, arches so designed have been primarily "dead load" structures, carrying relatively light live loads and resisting live load flexural stresses effectively by virtue of the heavy dead load compression in the rib. In such an arch, when built of the proper shape, the greater part of the stress is as definitely determinate as for a statically determinate structure. It seems scarcely necessary to add that this type of arch bridge is entirely different in essential structural characteristics from a structure of the same shape designed to carry relatively heavy live loads, particularly when the size of the rib is reduced by shifting material from the rib to the posts and floor.

The Final Report of the Special Committee of the Society on Concrete and Reinforced Concrete Arches[3] has called attention to some phases of the significance of interaction. For example, in Section I, Article 3, Conclusion 7 (*b*) suggests that live load stresses be determined considering the structure as a whole and not the rib alone, "because of the effect of the deck participation upon the stresses in the deck and columns rather than because of its effect upon the stresses in the rib"; and, again, Conclusion 9 points out that,

"A deck without intermediate expansion points, constructed so as to act with the rib in carrying loads, reduces the magnitude of the live load moment at the springing where the deck does not increase the strength of the structure; and it increases the moment over the central portion of the span where the strengthening effect of the deck is greatest."

There seem to be two not very clearly defined and sometimes conflicting points of view in the Committee's report and elsewhere in the technical literature on the subject of interaction: (1) That interaction is a source of danger and should be avoided if possible by means of joints and hinges or by other means of articulation; and (2) that interaction presents an opportunity to effect marked economies in design by taking advantage of its favorable aspects. This latter point of view is best shown by a review of a paper[4] which states, "French engineers have shown Americans that remarkably light and efficient concrete bridges can be designed by considering the spandrel columns and floors as integral with the ribs."

Previous Investigations.—Any of a number of analytical methods may be used for the determination of stresses in an arch with superstructure. Most of them were developed primarily for trusses without diagonals and for building frames. In general, they will be found to consist of variations in either of two fundamental procedures:

(1) Maxwell's procedure in which cuts or hinges are introduced at enough points in the structure to make it statically determinate and then forces are found to produce continuity.

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[3] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1427.

[4] "Integral Arch Action" by George E. Beggs, M. Am. Soc. C. E., on file in Engineering Societies Library; reviewed in *Civil Engineering*, December, 1933, p. 697.

(2) A method used by Hardy Cross,[5] similar to Ostenfeld's deformation method, in which various independent sets of joint displacements are produced and the forces involved are determined. These independent configurations are then combined in such proportions as to give the forces acting on the structure.

Several approximate analytical methods[6] have been suggested but are not generally available. Various simplifying assumptions permit a fairly rapid solution of the problem. The writer has made studies of different types of arch bridges using methods of analysis[7] that he has adapted, in order to test the validity of the simplifying assumptions that have been suggested. In general, none of the approximate solutions is applicable to a wide range of structural types. Exact methods of analysis require almost as great an expenditure of time and effort as that necessary for the completion of an accurate model analysis.

Analyses by means of models using the deformeter gage introduced by George E. Beggs[8], M. Am. Soc. C. E., and by models using the polarized light method[9] have been made. Deformeter model analyses are reported in articles describing the arch viaduct, at Ashtabula, Ohio,[10] the George Westinghouse Bridge[11], in Pittsburgh, Pa., and the Raritan River Bridge, in New Brunswick, N. J[12]. In these cases it should be noted that revision in design was not always made from the results of the analyses.

A model analysis was used to check a simplified theoretical analysis of the Grandfey Viaduct[13], in Switzerland. It is not clear from the article whether any advantage was taken of interaction in the design of the rib. From its size it would appear not. It may be noted that[14] "greater stresses are conceded for the superstructure than those for the main arches" due to the deformation of the arches resulting in a greater loading of the superstructure than that contributed by direct loading alone.

The experimental method may be summarized briefly as the determination of stresses in a prototype loaded in a manner similar to the actual structure.

[5] See, for example, "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., pp. 105, 228, and 236.

[6] "Statically Indeterminate Structures," by Hardy Cross M. Am. Soc. C. E., pub. in mineographed form, 1926, p. 167; see, also, "The Effect of Temperature Stresses in Open-Spandrel Arches of Reinforced Concrete," by M. F. Lindeman, Thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering, Univ. of Illinois, 1930.

[7] "Interaction Between Rib and Superstructure in Concrete Arch Bridges", by N. M. Newmark, Jun. Am. Soc. C. E., Thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, Univ. of Illinois, 1934 (prepared under the supervision of Professor Cross).

[8] "Model Analysis of a Reinforced Concrete Arch" (Yadkin River Bridge), by J. T. Thompson, M. Am. Soc. C. E., *Public Roads*, Vol. 9, No. 11, January 1929 p. 209; also, "Deck Participation in Concrete Arch Bridges," by A. H. Finlay, Assoc. M. Am. Soc. C. E., *Civil Engineering*, November, 1932, p. 685.

[9] "Cours de Beton Armé," by A. Mesnager, Paris, 1921, p. 223.

[10] "Features of an Open-Spandrel Arch Viaduct," by J. R. Burkey, *Engineering News-Record*, Vol. 101, No. 25 (December 20, 1928), p. 919.

[11] "A Concrete Arch of 460-ft. Span," by G. S. Richardson, M. Am. Soc. C. E., *Engineering News-Record*, Vol. 106, No. 17 (April 23, 1931), p. 680.

[12] "Two High Traveling Towers with Chutes Place Concrete on New Brunswick Multiple Arch Bridge," by Morris Goodkind, M. Am. Soc. C. E., *Engineering News-Record*, Vol. 104, No. 8 (February 20, 1930), p. 317.

[13] "Construction of the Grandfey Viaduct," by Adolph Bühler, *Proceedings*, Am. Concrete Inst., Vol. 23 (1927), p. 489.

[14] *Loc. cit.*, p. 526.

The most notable examples of experiments on arches with decks have been those conducted on the Yadkin River Bridge[15] and the series of experiments[16] by W. M. Wilson, M. Am. Soc. C. E., at the University of Illinois, on concrete models. It is important to note that in these experiments cracking and disintegration of the superstructure occurred in tests to destruction, the superstructure being affected almost to the same extent as the rib, although the loading was chosen so as to produce maximum stress in the rib.

Object of Present Study.—This paper relates certain phases of the behavior of arches with integral spandrel structure to the general philosophy of design presented recently by Hardy Cross[2]. It is not concerned with methods of analyzing stresses, nor with the details of design; it develops what is believed to be a rational point of view of the problem, and presents material leading to a possible basis for design.

The variables involved in this problem are numerous. The mere compilation of data, whether from experience, experiment, or analysis, does not always assist in clarifying a problem in which the variables are so numerous and difficult to separate; but with the classification of indeterminate structures developed by Professor Cross, one has available the basis for classifying and interpreting data bearing on the subject, or on related problems concerning other types of structures.

It is necessary to supplement experimental and analytical data with information pertaining to experience. What evidence, if any, is there of overstress in the floors, columns, or ribs of arches that have been constructed? Where have cracks occurred, and in what type of structure? That is, what are the relative stiffnesses of floor, columns, and rib, and where are the expansion joints (if any) located? The writer hopes that those who have information along these lines will contribute to the discussion. He would ask a further question: Has an arch bridge actually been designed and constructed taking interaction into account, and what changes in floor, columns, or rib were thereby made possible, or became necessary?

Interpretation of Structural Action

Classification of Indeterminate Structures.—In revising the design of a member in an indeterminate structure from the results of an analysis, it is necessary to know how the stress in the member will be affected by revision in its proportions, and by revision in the dimensions of other members in the structure. Professor Cross has defined qualitative types of structural action in terms of the response of the member to changes in design[2,17].

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[15] "Loading Tests on a Reinforced Concrete Arch," by A. L. Gemeny and W. F. Hunter, *Public Roads*, Vol. 9, No. 10, December, 1928, p. 185.

[16] "Laboratory Tests of Reinforced Concrete Arches with Decks", by W. M. Wilson, *Bulletin 226*, Eng. Experiment Station, Univ. of Illinois, 1931; "Laboratory Tests of Multiple-Span Reinforced Concrete Arch Bridges," by W. M. Wilson, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 424; and "Laboratory Tests of Three-Span Reinforced Concrete Arch Bridges with Decks on Slender Piers", by W. M. Wilson and R. W. Kluge, Jun. Am Soc. C. E., *Bulletin 270*, Eng. Experiment Station, Univ. of Illinois, 1935.

[17] *Proceedings*, Am. Soc. C. E., April, 1936, p. 536.

Briefly, the following types of action are defined, resulting either from external fixed loads or from imposed fixed deformations on the structure:

(1) Participation defines the action in which non-load-carrying or nuisance stresses are developed in a member due to very nearly fixed strains resulting from the deformation of the load-carrying members;

(2) Normal action corresponds to a very nearly fixed magnitude of the forces and moments acting on a member, and is developed when the forces acting on the member are practically independent of the size of the members in the structure; and,

(3) Hybrid action is best defined as action in a range intermediate between participation and normal action, where forces, moments, and strains are dependent on the size of the member considered, or on the proportions of other members in the structure.

It follows that, in redesigning a member found by analysis to be overstressed, participation stress can only be lessened by reducing the strain the member must undergo; for example, linear strain due to angle changes is easily reduced by decreasing the depth of the member. Normal stress can be reduced by increasing the size of the member, but hybrid stress may be either increased, decreased, or unchanged when the size of the member is increased. Where maximum stress arises from a combination of circumstances it is necessary to keep in mind the relative importance of the different factors contributing to the stress in a member in arriving at a design.

Load-Carrying and Nuisance Stress.—In considering the significance of over-stress, it is convenient to make a distinction between stresses depended upon to "carry the load" and secondary or nuisance stresses resulting from the action of the structure but having no useful function. Over-stress in normal action is serious. Over-stress due to participation is dangerous only to the extent that the ability of the member to perform its primary function is impaired. Over-stress in hybrid action may or may not be serious depending on whether the structure may "carry the load" safely in another way when the over-stress becomes large. In this connection, the significance of chance factors in the properties of the material becomes important.

The fact that the superstructure of an arch bridge is slightly overstressed is not always serious if the superstructure is not depended upon to reduce the stress in the rib. If it is possible to reduce the size of the rib of an arch bridge because of interaction, but if at the same time the superstructure must be strengthened and if the resulting structure is neither appreciably more economical nor certainly more safe than the structure as designed neglecting interaction, then the process is futile.

Some Phases of Interaction Between Rib and Spandrel Structure

Results of Analyses; Moment Diagrams.—The writer has made analyses of different arch structures but, for the reasons stated previously, finds it difficult to draw empirical conclusions concerning interaction from the results of

these analyses. However, some of the data obtained are of interest in showing, qualitatively, the effect of interaction, and are presented for that purpose.

One of the series of arches analyzed consisted of a set of five-panel structures having different decks, all with practically the same rib. Sketches of the structures are shown in Fig. 1. The rib is parabolic in shape with the moment of inertia varying directly as the secant of the angle of inclination with the horizontal. Several different values of moment of inertia of deck and columns were considered. In Fig. 1, all the structures are symmetrical about the center line. The dimensions denote relative distances and the relative moment of inertia is given as I. The structure with a high deck is shown in Fig. 1(a), the structure with a low deck and an infinitely stiff saddle at the crown is shown in Fig. 1(b), and the structure consisting of a flexible polygonal rib with a stiffening girder is shown in Fig. 1(c). Except for the saddle, the rib in Fig. 1(b) is the same as that in Fig. 1(a).

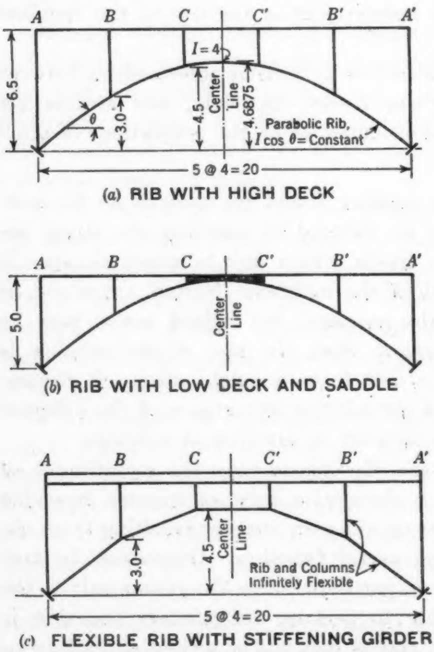

FIG. 1.—TYPES OF ARCH STRUCTURES ANALYZED.

Consider the particular structure shown in Fig. 2(a) loaded with a force, P, and having reactions, R. Let H denote the horizontal component of the reactions, and let V denote the vertical components. At any vertical section through rib and deck, such as y-y, the sum of the horizontal components of the forces in rib and floor must equal the horizontal component of the reactions:

$$\text{Total } H = H \text{ in rib} + H \text{ in floor} \dots\dots\dots\dots\dots (1)$$

Furthermore, the sum of the vertical components of the forces in rib and floor must equal the total vertical shear, V, at the section:

$$\text{Total } V = V \text{ in rib} + V \text{ in floor} \dots\dots\dots\dots\dots (2)$$

Curves of V and H are shown in Fig. 2(a) with the numerical values of the total forces given on the diagram. The part of V and H resisted in the floor is shown by the stippled areas, and the remaining areas denote the part of V and H resisted in the rib. The change in the part of H resisted in the floor at the column points represents the shear in the column.

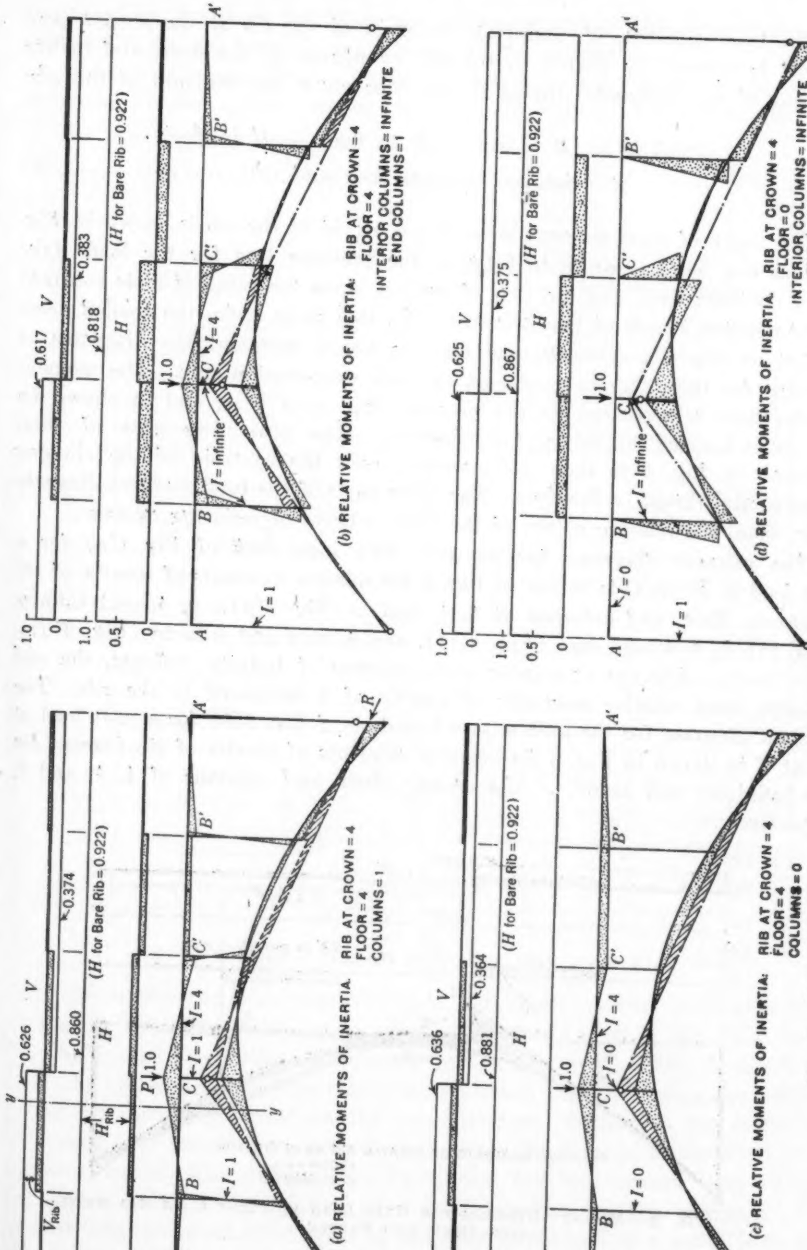

FIG. 2.—MOMENT DIAGRAMS FOR A UNIT LOAD AT POINT C ON RIB WITH HIGH DECK.

The total moment, M, about the centroid of the rib at the section considered is resisted by flexure in the rib, by flexure in the floor, and by the moment of the horizontal thrust in the floor about the centroid of the rib:

$$\text{Total } M = M \text{ in rib} + M \text{ in floor} + H \text{ in floor} \cdot \times \text{(distance between floor and rib)} \quad (3)$$

The curve of total moment about the centroid of the rib is shown in Fig. 2(a) as the vertical ordinate between the pressure line for the loads (the heavy dot-dash line) and the axis of the rib, since the value of H is constant for the entire length of the structure. To this same scale, the shaded areas plotted on the rib, columns, and floor as bases, represent the diagrams of moment for these members, plotted on their compression side. The moment in the floor is replotted on the pressure line as a base, and is shown by the cross-hatched areas in the diagram. The remaining area of total moment in Fig. 2 is then the moment about the centroid of the rib due to horizontal thrust in the floor. The three parts of the total moment diagram show, then, the relative values of the three sources of resisting moment.

The moment diagrams for the arch with high deck of Fig 1(a) for a unit load at Point C is shown in Fig. 2 for relative moments of inertia of rib at crown, floor, and columns of 4, 4, and 1 (Fig. 2(a)), 4, 4, and infinite (Fig. 2(b)), 4, 4, and zero (Fig. 2(c)), and 4, zero and infinite (Fig. 2(d)), respectively. For the structures with columns of infinite stiffness, the end columns have relative moments of inertia of 1 compared to the rib. The moment diagram for the arch with a low deck of Fig. 1(b) for a unit load at Point C is shown in Fig. 3 for relative moments of inertia of rib (except for the infinitely stiff saddle at the crown), floor, and columns of 4, 4, and 1, respectively.

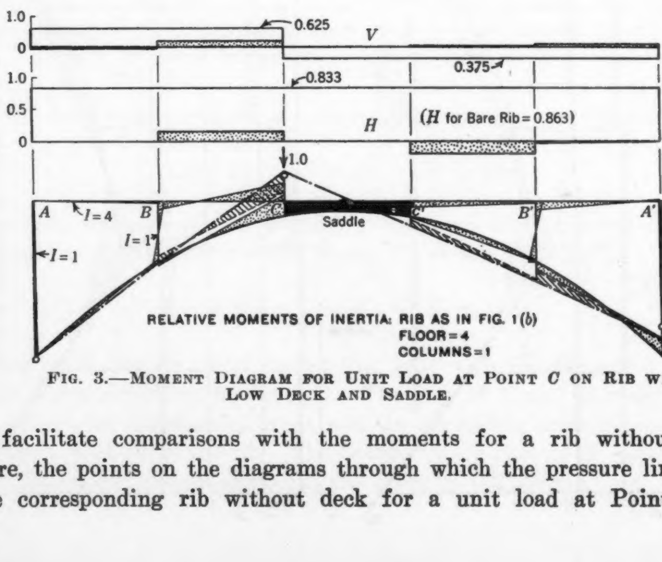

FIG. 3.—MOMENT DIAGRAM FOR UNIT LOAD AT POINT C ON RIB WITH LOW DECK AND SADDLE.

To facilitate comparisons with the moments for a rib without superstructure, the points on the diagrams through which the pressure line passes for the corresponding rib without deck for a unit load at Point C, are

indicated by the small circles at the springing and at the load point. The value of H for the bare rib is indicated on the diagrams.

Typical Influence Lines.—Typical influence lines for moment and flexural stress in rib and superstructure are shown in Fig. 4. They were computed

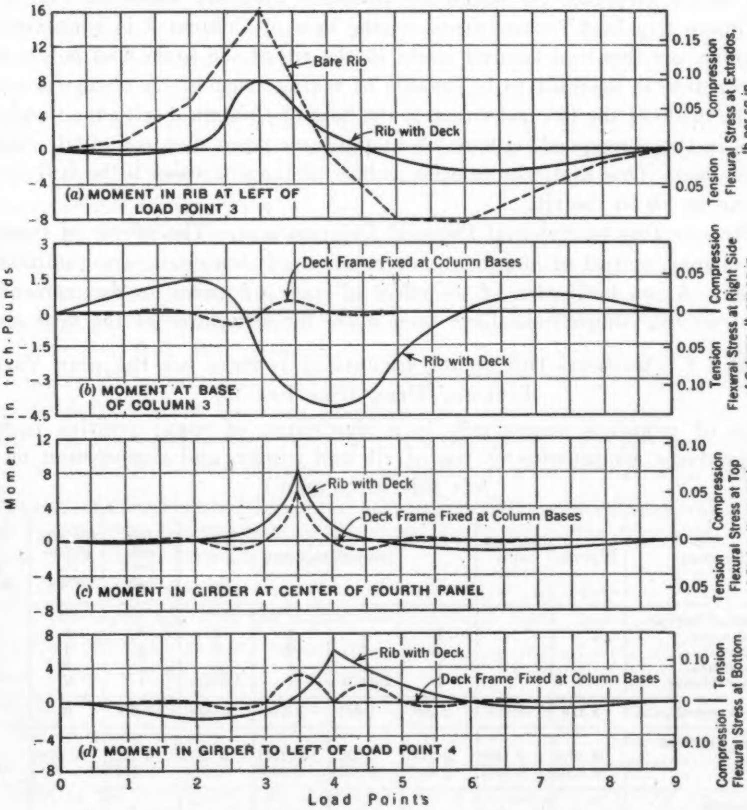

FIG. 4.—TYPICAL INFLUENCE LINES FOR ARCH WITH HIGH DECK.

for the single-span structure having a high deck without expansion joints tested by Professor Wilson, a drawing of which is shown in the Final Report of the Special Committee on Concrete and Reinforced Concrete Arches.[18]

The broken lines in Fig. 4(*b*), Fig. 4(*c*), and Fig. 4(*d*) show the influence lines for "primary stress" in the superstructure, defined as the stress with the deck and columns considered acting as a continuous frame fixed at the column bases. The differences between the full and broken lines in these diagrams indicate the relative effect of interaction on the moments. It is noted that the interaction effect in the floor at the end of a panel is so great that the sign of the moment for live loading is changed from that corresponding to primary action.

[18] *Transactions*, Am. Soc. C. E., Vol 100 (1935), p. 1486, Fig. 41(*a*).

The scales for flexural and direct stress shown in the diagrams are computed on the basis of an uncracked concrete section, neglecting the steel reinforcement. The values obtained show the relative effects of moment in the various members considered. It appears that for a single concentrated load on the structure for which the influence lines are shown in Fig. 4, the maximum live load flexural stress at the base of Column 3 is approximately 1.4 times the live load flexural stress in the rib at the same load point, when the concrete is assumed to be capable of taking tension. A comparison may also be made of the flexural stress at the base of Column 3 with the maximum stress in the corresponding bare rib at the same point due to a single concentrated load. One finds the relative values of these stresses to be 0.67 in the column to 1.0 in the rib.

Stresses Due to External Imposed Deformations.—The effects of temperature change, spread of abutments, or rotation of abutments, are qualitatively similar. As an indication of the effect of such influences on the moments in the structure, comparisons have been made for structures of the type shown

TABLE 1.—MOMENTS DUE TO A UNIT CHANGE IN SPAN FOR RIB WITH VARIOUS TYPES OF DECK (SEE FIG. 1).

(Sign of moments corresponds to a shortening of span; positive moment produces compression at top of rib and girder, and compression on left side of column.)

Structure	Bare rib	Bare rib with saddle	Rib with high deck				Rib with low deck	Rib with stiffening girder	Stiff rib with girder
Moments of Inertia:									
Rib at crown	4	*	4	4	4	4	*	0	4
Girder	....	....	4	4	0	0	4	4	4
End columns	....	....	1	1	1	1	1	0	0
Interior columns	....	....	1	Infinite	1	Infinite	1	0	0
Height to neutral point	3.125	2.750	2.290	1.451	2.914	1.826	2.278	0	2.814
Springing Reactions:									
H	0.1024	0.1439	0.2134	0.5224	0.1341	0.3662	0.2461	0.0183	0.1384
M	0.3200	0.3957	0.4887	0.7578	0.3908	0.6685	0.5607	0	0.3894
Rib Moments:									
M_{AB}	0.3200	0.3957	0.4552	0.7363	0.3554	0.6330	0.4949	....	0.3894
M_{BA}	0.0103	−0.0360	−0.1303	−0.5310	−0.0306	−0.4491	−0.1611	....	−0.0532
M_{BC}	0.0103	−0.0360	0.0045	0.4526	0.0453	0.5091	0.0029	....	−0.0532
$M_{CB'}$	−0.1408	−0.2518	−0.1335	−0.2804	−0.1802	−0.4426	−0.1845	....	−0.0983
$M_{CC'}$	−0.1408	....	−0.0242	0.1732	−0.0960	0.1124	....	....	−0.0983
M_{cr}	−0.1600	....	−0.0980	−0.0901	−0.1321	−0.0586	....	....	−0.1242
Girder Moments:									
M_{AB}	....	....	0.0040	0.0279	0	0	−0.0116	0	0
M_{BA}	....	....	−0.0371	−0.2749	0	0	−0.0475	−0.0548	0.0274
M_{BC}	....	....	0.0800	0.3304	0	0	0.0843	−0.0548	0.0274
$M_{CB'}$	....	....	−0.2033	−0.4026	0	0	−0.2961	−0.0822	−0.1351
$M_{CC'}$	....	....	−0.0872	−0.0023	0	0	....	−0.0822	−0.1351
M_{cr}	....	....	−0.0872	−0.0023	0	0	....	−0.0822	−0.1351
Column Moments:									
Top: M_A	....	....	−0.0040	−0.0279	0	0	0.0116	....	0
M_B	....	....	−0.1171	−0.6054	0	0	−0.1318	....	0
M_C	....	....	−0.1161	−0.4002	0	0	....	....	0
Base: M_A	....	....	−0.0335	−0.0216	−0.0355	−0.0355	−0.0658	....	0
M_B	....	....	0.1348	0.9836	0.0759	0.9583	0.1640	....	0
M_C	....	....	0.1092	0.4536	0.0842	0.5550	....	....	0

* Infinitely stiff saddle at crown; otherwise, rib same as bare rib.

in Fig. 1. Data are shown in Table 1 for springing reactions and moments in all members due to a unit change in span for an arch rib with various types of deck. The effect on the rib of interaction with the deck is to increase the temperature moments considerably.

Consider the deformations of an arch structure with deck due to a given change in span. If the columns have little stiffness and the floor, at all sections, has the same relative stiffness to that of the rib, the change in shape of the rib will not be altered by the superstructure. Consequently, in such a structure, the temperature flexural stress in the rib will be the same as in the bare rib. However, there will be temperature flexural stresses in the floor, which will be to the temperature stress in the rib at any section as the depth of the floor is to the depth of the rib at that section, since the deflections (and, consequently, angle changes), of the rib and floor are the same. Note that the direct temperature stresses will be higher in the composite structure than in the free rib, but the direct stresses are comparatively insignificant.

If the floor has, say, a constant moment of inertia and that of the rib varies, and the columns still have very little stiffness, the change in shape of the rib due to a given abutment movement will be altered somewhat from the change in shape of a free rib since the composite structure is relatively more stiff at the crown than at the springing compared with the free rib. Hence, there will be a greater distortion in the haunch and near the springing with a relatively higher temperature stress in the composite structure at those points than in the bare rib.

Consider now the effect of the columns. Since, in general, the columns stiffen the structure and the given displacement must take place, there will be correspondingly greater distortions in the rib with stiffer columns; and, hence, there will be greater deformation stress in rib and floor with stiffer columns. The columns also will have high moments due to deformations.

The increase in relative stiffness of the crown of the composite structure over the crown of the free rib causes a reduction in maximum live load moment at the springing, but this same cause produces inevitably an increase in temperature flexural strain at the springing. The fact that one effect cannot exist without the other is worth some emphasis.

The significance of high deformation stresses is a matter of judgment. They are almost certainly not as serious in concrete as stresses due to loads. Attempts have been made to reduce stresses due to these causes by expansion joints and other articulations in the deck of the bridge. These attempts are not always successful, sometimes merely localizing the effect, particularly in the case of expansion joints at the ends of a saddle in a low deck bridge.

It must be noted that for structures in which interaction is depended upon to reduce stresses in the rib the stresses in the deck due to deformations and temperature changes are important in so far as they tend to cause cracking of the superstructure, for cracking of the superstructure reduces its stiffness and thereby the aid which it can render the rib.

Summary of Results of Analyses.—A study of a number of influence lines and moment diagrams seems to indicate the following conclusions:

(1) The maximum live load moment at the springing line and at intermediate column points on the rib of the integral structure is considerably reduced by the interaction with the deck. For intermediate column points the reduction may be of the order of 50% when the moment of inertia of the floor is equal to that of the rib. The moment between column points in the rib is reduced considerably more than the moment at the column points if the columns are stiff.

(2) The interaction between rib and deck for live load adds to the primary moment in the floor at the center of the panel, computed as for a frame fixed at the column bases. At the ends of the panels, the interaction moment in the floor is opposite in sign to, and may be much greater than, the primary moment.

(3) The interaction live load moments in the columns are considerably greater than the primary live load moments, even for slender columns. The interaction column moments are greatly increased by making the columns stiff.

(4) Where the columns are quite stiff and the rib and floor have approximately equal moments of inertia, points of inflection or points of zero moment are close to the center of all members for most conditions of loading.

(5) Temperature and deformation strains at the springing in the rib of the integral structure are much greater than in a bare rib. Temperature strains in the deck are likely to be of the same order as those in the rib. The temperature strains are increased by making the columns stiff.

Treatment of the Arch with Superstructure as a Flitch-Beam.—If the spandrel columns of an arch bridge are quite flexible and closely spaced, it is evident that the deflections, slopes, and angular changes along the arch rib must be equal to those along the floor (except for the change in length of the columns, which effect will be neglected, but can be taken into account if necessary). Then all direct stresses due to thrust will be carried by the rib, and the relation between moments and flexural stresses in rib and floor can be predicted and controlled by varying their relative depths and moments of inertia.

In other words, the structure can be designed as if it were composed of two members placed side by side as in a flitch-beam. This concept dates back at least to Rankine,[19] and has been used apparently by Considére.[20] Professor Cross[2] has discussed several new aspects of this treatment in connection with his system of classification.

An analysis for the purpose of design may be developed along the lines of Professor Cross' treatment as follows: The deck girder will have primary

[19] See "Civil Engineering," Seventh Edition, 1871, pp. 313–314; and other editions at approximately the same page numbers.

[20] See, for example, "Cours de Beton Armé," by A. Mesnager, Paris, 1921, p. 198.

stresses due to its action as a continuous beam. These stresses are due to the dead load of the deck only, and the entire live load. The arch rib will take all the thrust, most of which is due to dead load. Then the difference between the working stress and the primary girder stress is available to resist interaction flexure in the girder. The difference between the working stress and the stress due to direct thrust in the arch rib is available to resist flexure in the rib. Temperature and deformation stresses may also be discounted from the working stress.

The stress due to interaction flexure in the girder is to the flexural stress in the rib approximately as the depth of the girder is to the depth of the rib. The rib and deck resist the moment (except for primary moments) approximately in proportion to their moment of inertia.

A few rough figures will enable an estimate to be made as to whether a tentative combination of depths of girder and rib is efficient or inefficient, and will give some idea as to what can be done about it; for example, if the girder is found to be overstressed, it must be remembered that increasing the depth of the girder may increase interaction stress, although it decreases the primary stress.

Analyses of several structures by the writer have indicated that for loads applied at column points, the approximation involved in the assumption that the columns are closely spaced is very good. This approximation is strictly true for the particular case of a two-hinged arch rib polygonal in shape with a deck composed of straight prismatic segments between column points, in which the ratio, $\frac{(E\,I \cos\theta)_{\text{deck}}}{(E\,I \cos\theta)_{\text{rib}}}$, has the same value in every panel. For loads between column points it is necessary, for accuracy, to find, first, the column reactions, considering the floor as a continuous beam, and then to apply these reactions to the composite structure.

Where there are expansion joints in the floor the total moment must be taken by the rib. Where the floor is not continuous over the springing the moment at the springing is resisted entirely by the rib.

For certain cases it is possible that neglect of the columns will not be on the safe side. If the columns are stiff and the floor flexible there is considerable local flexure of the floor and the moments in the floor may be several times as great as those assumed on the basis of equal angle changes, unless the stiffness of rib and floor is of about the same order.

Relative Importance of Sources of Stress.—The principal sources of stress for which an arch bridge is designed are dead load, live load, and deformations such as those due to temperature change, shrinkage, abutment movements, etc.

Neglecting the effect of rib-shortening there will be no interaction stress for dead load if the pressure line for the weight of the rib and the dead load reactions of the deck acting as a continuous frame fixed at the column bases

fits the arch axis. With some reservations,[21] rib-shortening may be considered as deformation stress. The manner of constructing the structure must be considered in evaluating the effect of rib-shortening; for example, if the arch centering is struck before the deck is constructed, rib-shortening due to the weight of the rib will not cause interaction stress.

The dead load and live load moments in the floor considered acting as a frame fixed at the column bases (defined as the primary moments in the floor) depend mainly on the panel length. The interaction moments in the floor are a function of the relative stiffness of floor and rib as well as of the span length of the rib. Adjustments of the depth of floor and rib, therefore, may make the interaction stress in the rib almost anything that is desired within reasonable limits. If it is desired to reduce interaction stress in the floor to a minimum, it is most easily done by reducing the depth of the floor by a proper choice of details and arrangement of framing.

For very short arches, say, spans up to 50 ft perhaps, the dead load stress is relatively small compared to the live load stress for an open spandrel arch, but such short spans are usually constructed more economically with filled spandrels. For arches of perhaps 150 ft, or more, in length, the dead load is generally more important than the live load. For long-span bridges interaction has no effect on the major part of the stress in the arch rib. In this connection it is interesting to note what has been written[11] concerning the design of the George Westinghouse Bridge of 412-ft span length:

"A model study of the main span was made with the Beggs Deformeter to determine the stiffening effect of the superstructure. This study showed that the negative live load moment at the springing line would not exceed 60 per cent of the moment calculated for the bare rib. When this moment is reduced to terms of fiber stress the reduction is only from 780 to 730 lb. per sq. in."

A description[12] of the Raritan River Bridge, in New Brunswick, N. J., with a span length of 202 ft, contains the following comment:

"Each of the main arch spans consists of three hingeless ribs, 12 ft wide, designed as free ribs by the application of the elastic theory without taking into account the restraining action of the superstructure. The design was checked by the use of deformeter gages on a celluloid model, and the agreement with the analytical method was practically exact. A determination of the stresses in the rib was also made on a complete model of the rib and superstructure, and marked reductions in the moments, thrusts and shears were noted due to the stiffening effect of the spandrels and floor. However, since the superstructure was poured after the centering was struck, it does not relieve the rib of any of its dead load stresses. The braced condition of the rib is applicable under live load, but as the live-load stresses comprise but a small proportion of the total, no advantage was taken of the reduced stresses in a redesign of the rib."

In even shorter arch bridges it may be noted that a decrease in live load stress in the rib is to some extent offset by an increase in deformation and temperature stress.

[21] "Relation of Analysis to Structural Design," by Hardy Cross, *Proceedings*, Am. Soc. C. E., October, 1935, p. 1119, especially p. 1121.

Studies made of arch bridges in Allegheny County, Pennsylvania, for both springing and crown stresses for spans between 180 ft and 400 ft, led to the following conclusions[23]:

"For the extreme condition considered, the moment due to temperature and shrinkage was doubled to indicate the possible effect of additional moment from any cause such as a temperature drop or shrinkage greater than the assumed or movement of the supports. This investigation leads to interesting conclusions. Considering the extreme condition of combined stress, which gives an index to the ultimate safety of the rib, it develops that:

"(1) The live load stress is a relatively small proportion of the total ranging from about 15 per cent for spans approximating 400 ft. to about 25 per cent for short spans. These percentages apply to shallow ribs, and are smaller for deep ribs.

"(2) The arch shortening stresses may become more than 50 per cent of the total for deep ribs."

The stresses in the columns cannot be estimated very closely. The columns are ordinarily not designed for moment in any case, and usually not even for direct stress. Their least dimension depends upon their height, and when all the columns are made the same width, this dimension depends on the height of the longest column. The columns may possibly be considerably overstressed in flexure due to interaction. This is not necessarily serious unless the columns are depended upon to reduce the stress in the rib.

Various studies made by the writer seem to indicate that the column action is generally always hybrid. The effect on the column moments of increasing the size of the column, is demonstrated in Figs. 2 and 3. Moreover, even when the columns are so stiff that further increase in their section does not attract more moment to them, the column moments are still sensitive to variations in the ratio between rib and floor stiffness.

Classification of Arches with Integral Spandrel Structure

Rib with Flexible Deck.—If the superstructure is very flexible compared with the rib the interaction stresses, due to movement of the column bases, will be small compared with the primary stress in the deck, considering the column bases fixed. This type of structure is clearly normal with participation stress in the deck.

It is worth while to examine the possibility of reducing interaction stresses by detailing the superstructure so that it has a high degree of flexibility. Jack or spandrel arches and fascia beams may be omitted and the slabs framing into the floor-beams at each panel may be used to secure flexible construction. A structure so designed may possibly be overstressed in the deck, but the over-stress will not be serious since the deck is not depended upon to carry load. Any cracking in the deck will relieve the stress, but will not cause over-stress in the rib because the rib is proportioned to carry the entire load.

A method for determining interaction stresses in a structure of this kind has been suggested by Hardy Cross.[6] The method is akin to that com-

[23] "The Design of Concrete Arches in Allegheny County, Pennsylvania," by G. S. Richardson, M. Am. Soc. C. E., *Proceedings*, Am. Concrete Inst., Vol. XXVII (1932), p. 650.

monly used to determine secondary stresses in bridge trusses and is as follows: For the given condition of loading determine the deflections of the column points in the arch bridge. In general, these deflections will be a horizontal and a vertical translation and a rotation. For these given deflections find the stresses in the superstructure. If the forces at the column base are small compared to the loads acting on the arch, no revision is necessary. If they are not negligible a revision may be made by computing the additional deflection of the arch rib due to these forces and revising the figures. If this revision is large, the method will not work. It may be used, however, to determine whether or not the secondary stresses are serious in any particular case.

The flexibility of the deck depends mainly on the panel length, and it is evident that decks of the same type and panel length in structures of different length will be practically the same in stiffness. The longer the arch bridge, the stiffer will be the rib. For structures of short and moderate lengths the deck, of necessity, will be fairly stiff as compared to the rib. The flexible deck type of structure will usually be found, therefore, in long spans.

The Flitch Arch.—If it is necessary or seems desirable to make the floor fairly heavy, it is possible to make use of the flitch-beam concept in the design of the structure, and depend on the floor, but not on the columns, in interaction. Where both the rib and the floor are depended on to carry the load, the structure is hybrid, but the controlling relations are comparatively clear; the designer can "tell the structure what to do" with reasonable assurance that it will act in the way he designs it.

The particular case of a flexible flitch arch with stiffening girder is of interest. Several arches of this type have been built in Switzerland, Sweden, and Finland. The Landquart Bridge, in Switzerland, is of this type, has a polygonal rib about 10 in. deep connected by flexible columns to a floor supported by girders approximately 3 ft in depth. Stresses in this type of structure may be developed directly from the flitch-beam concept by considering a two-hinged arch rib having the shape of the given rib and a moment of inertia variation, in which $\frac{ds}{I}$ is equal to $\frac{dx}{I}$ for the girder. The analogy holds for flexural stresses due to all causes, including rib-shortening. Of course, all direct stresses are carried by the arch rib, including direct stress due to rib-shortening and deformations. The secondary flexural stresses in the rib may be obtained from the relation that the flexural stress in the rib is to the interaction flexural stress in the girder approximately as the depth of the rib is to the depth of the girder. This type of flitch arch is a normal structure with participation stresses in the rib adding to the primary stress due to thrust.

The flitch-beam concept may be applied to the structure with a saddle connecting the deck and rib, providing the columns outside the saddle are fairly flexible. The section through the saddle may be considered as having

an infinite moment of inertia. It should be pointed out that stresses in such a saddle, even for given moments, shears, and thrusts, cannot be computed accurately by methods of elementary mechanics. Usually, it will not be necessary to compute the stresses since the resistance of the saddle to flexure is very high. If the deck outside the saddle is quite flexible it may be convenient to consider the section common to rib and deck as a part of the rib alone, and neglect any other effect of the deck. The method of computing the participation stress in the floor suggested in the preceding section may then be used conveniently.

The flitch-beam concept applies also to the through arch bridge having closely spaced steel hangers supporting the roadway. Where the roadway is used as a tie, the procedure with slight modifications may still be used. If the tie and the roadway have no flexural resistance the structure is what is ordinarily known as the bowstring arch, which may be analyzed conveniently by the usual methods. The secondary flexural stresses in the roadway may be approximated as shown previously for similar structures.

There is nothing in the flitch-beam concept requiring that the structure be symmetrical. Unsymmetrical arches may be treated in the same manner as symmetrical arches.

Arch with Stiff Spandrel Columns.—The structure in which the effect of the columns is important is, in general, in the hybrid class. There are certain exceptions. If the columns are quite stiff, and if the rib and floor have approximately equal moments of inertia, the structure has certain normal characteristics. The corresponding simply supported Vierendeel truss is a normal structure and can readily be designed. For the Vierendeel truss an approximate analysis preliminary to design may be made by assuming points of contraflexure in the center of all members and also that at any section the moments in rib and floor are equal. For the arch, it is necessary to take into account the fact that the supports are fixed. It is noted in this connection that the arched structure is hybrid to the same extent that a bare arch rib is hybrid for live loads.

For the general case of the structure with moderately stiff columns, where the rib and floor are quite different in stiffness, the structure is certainly hybrid. It may be possible in certain cases to choose, arbitrarily, proportions which analysis will prove to be fairly satisfactory. In general, however, no definite rules can be given for the revision of such a design from the results of an analysis; nor is the stress in such a structure known certainly, even for given dimensions, because of the effect of variations in physical properties of the different members. The hybrid structure, particularly the complicated hybrid structure of this type, may be quite sensitive to variations within itself. It may be designed, of course, by choosing to neglect the uncertain factors. In any case it must be ascertained whether the neglect of these factors is on the safe side if any advantage is to be taken of interaction in the design.

EFFECT OF EXPANSION JOINTS AND OTHER TYPES OF ARTICULATION

The articulation of the structure by means of expansion joints in the floor and hinges, rollers, and sliding joints in the columns, affects principally the columns. As far as the rib is concerned, due to articulation of the deck, the dead load stress is unaffected, the decrease in live load stress over that for the plain rib is less, and the increase in temperature stress at the springing over that for the bare rib is less, than for the structure without articulation. In the floor both the interaction live load stress and the temperature stress are decreased by articulation of the floor, but in the case of the live load stress the decrease is not appreciable several panels away from the articulation.

Live load stresses in the columns are not always decreased by expansion joints in the floor. The expansion joint tends to localize the rib deformation, and, as a consequence, the neighboring columns may suffer. Temperature stresses in the columns are probably reduced in all cases by expansion joints because of the decreased stiffness of the entire structure.

Articulating the columns rather than using expansion joints in the floor is more effective in reducing the column moments, but the details of such articulations usually become quite troublesome.

SUMMARY

The most important points to be emphasized in this paper may be summarized as follows:

(1) The net effect of interaction on the total stress for which an arch rib must be designed is normally quite small. The ordinary method of designing arch structures neglecting interaction is sound, in general, since structures so designed have apparently given good service.

(2) The significance of over-stress in the superstructure depends upon whether the deck is counted upon to assist the rib in carrying the load. It may be possible to obtain greater economy by detailing floor and columns to reduce their rigidity, and hence their interaction stress, than by designing the superstructure to assist the rib. The conclusion that the deck always strengthens the rib must not be drawn too hastily. The possibility of a load cracking the deck and destroying part of the interaction must be considered. Furthermore, stresses due to temperature and movements of foundations are high in the integral structure.

(3) Certain limiting cases of the arch with integral spandrel structure are "normal," and can be designed directly, but practically all such structures of the proportions usually built are hybrid, and cannot be designed for given working stresses by the traditional procedure of repeated analysis and design.

(4) An arch with a comparatively heavy rib and flexible columns and deck supporting relatively light live loads is a normal structure with participation stress in deck and columns added to the normal stress in these members due to their action in the continuous deck frame.

(5) Another "normal" type is the structure consisting of a flexible rib with a stiffening girder. There will be flexural participation stress in the rib, in addition to the normal stress due to the direct thrust in the rib.

(6) A third type of structure having "normal" characteristics in certain cases may be described as an arched Vierendeel truss. This structure must have quite rigid columns, and floor and rib of approximately equal stiffness. The corresponding simply supported Vierendeel truss would be normal, but fixing the supports introduces complications. In this connection, it is important to note that the fixed arch rib has hybrid characteristics for live loads.

(7) Other "normal" arch structures may be secured in various ways by the manner of construction or design involving the use of articulations and flexible sections. Such structures have well marked participation stresses at such articulations and pseudo-hinges. Of course, not all structures with hinges and joints are "normal."

(8) In special cases applying to certain other arch structures which have hybrid characteristics the controlling relations are comparatively clear. The flitch-beam concept applying to the structure with flexible columns has considerable value and leads to a definite design procedure.

(9) For the general case where the columns must be of intermediate stiffness the structure is clearly hybrid, and there seems to be no convenient way of estimating in advance whether it can act as it is assumed to act in a preliminary analysis. Use of the traditional procedure of successive analysis (usually by means of a model) and design does not appear to be very promising. Such a procedure might easily be misleading, particularly where a re-design is based on an analysis which is not repeated for the new design. It is open to question whether a better structure is obtained by such a procedure than by neglecting interaction entirely.

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Founded November 5, 1852

DISCUSSIONS

INFLUENCE OF DIVERSION ON THE MISSISSIPPI AND ATCHAFALAYA RIVERS

Discussion

BY E. F. SALISBURY, M. AM. SOC. C. E.

E. F. SALISBURY,[21] M. AM. SOC. C. E. (by letter).[21a]—The frank criticism and constructive comments in the discussions are appreciated. The discussion by Professor Lane assists materially in clarifying the paper and the writer is practically in complete agreement with his views.

Professor Lane points out that, prior to the removal of the rafts from the Atchafalaya River, probably the greater portion of the flow of the Red River reached the Mississippi River and that now (1936) only a very small part of it reaches the main river. This diversion to the Atchafalaya of a greater part of the flow of the Red River may have much more effect on the channel below Red River Landing than the flow that passes out of the Mississippi through Old River.

The writer is not completely in accord with this view. If, during the period of this analysis, increasing volumes of water from the Red River were being diverted through the Atchafalaya, a resulting detrimental effect on the main channel is not questioned. However, although the Atchafalaya River has greatly increased in cross-sectional area, during the period of this study, it has not increased materially in discharge capacity; therefore, the conditions controlling the draw from the Red River have not changed during the time under consideration. The removal of the rafts in the Atchafalaya began about 1840 and was completed in the early Sixties; so there were from 20 to 40 yr for any channel adjustment before the earliest date used in the paper if the removal of the rafts would have the effect of drawing off more of the Red River discharge through the Atchafalaya than formerly. Consequently, it is believed that deterioration in the main channel below Red River Landing is due solely to diversion of water from the Mississippi through Old River.

NOTE.—The paper by E. F. Salisbury, M. Am. Soc. C. E., was published in November, 1935, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: April, 1936, by Leo M. Odom, Assoc. M. Am. Soc. C. E.; and May, 1936, by E. W. Lane, M. Am. Soc. C. E.

[21] Chf. Engr., L. & A. R. R., Minden, La.

[21a] Received by the Secretary August 24, 1936.

In commenting on the slope of the Atchafalaya River, Professor Lane, states in part:

"Although it seems improbable that the value of C has changed in the Mississippi, it is by no means improbable in a river scouring as actively as the Atchafalaya was during the period under consideration. The irregular cutting of the banks and bed which occurred under such circumstances might easily cause an increase in roughness. The great turbulence of the river is very noticeable at high flows and the computed roughness factors show considerably higher values than an ordinary river channel."

For some time the writer has been of the opinion that when the silt load of a heavily laden stream is subjected to a major disturbance, causing either a considerable increase or a considerable loss in the silt load, such as that produced by a diversion of the flow, a material increase in roughness factor is immediately produced over that customarily used for ordinary river channels. Definite proof as to this thought is not available at this time. The observations of the effects at the cut-offs being made in the Mississippi River may produce valuable data in this direction.

Mr. Odom is correct that the reference to the Guglielmini hydraulic theory as mentioned in the Humphreys and Abbot report on the "Physics and Hydraulics of the Mississippi River" is an "extract from the writings of Major J. G. Barnard, Corps of Engineers, U. S. Army." The writer is indebted to Mr. Odom for clearing up this reference.

In preparing the paper it was the writer's intention to interpret correctly that which the Mississippi River is recording in so far as the data applies to the subject treated. To the writer's knowledge no data have been intentionally selected or omitted. With this premise the data presented cannot be considered as selected as was intimated by Mr. Odom. On the basis of interpreting that which the river has recorded, it is premature to conclude that the "Synopsis" should include cut-offs as well as diversion. The further recordings of the river will be the answer to this separate problem. The reasons advanced by Mr. Odom as to the existence of a compact, and not readily scoured, clay in the bed of the Atchafalaya and Mississippi Rivers, are generally accepted. Furthermore, they apply to other silt-bearing streams and canals under certain conditions.

Below Red River Landing, the Carrollton gauge is the only recorded gauge and discharge study that provides data for analysis. Mr. Odom's statement that "at Carrollton the gauge height for bank-full discharge has not varied appreciably in the last 45 yr", depends upon the limitations as defined by the word, "appreciable." That the gauge has increased for the same volume of water is apparent from the gauge study. The increase in gauge for the same volume of water at Carrollton could not be expected to be of the same magnitude as that at Red River Landing as it is about 191 miles down stream on the slope. Furthermore, it could be expected that the gauge increase at Baton Rouge for the same volume of water will be less than at Red River Landing as it is about 60 miles below, and yet greater than at New Orleans, which is 124 miles farther down stream. Discharge observations are not taken in the Mississippi River at Baton Rouge. To study the action of the river at this

point, gauge heights were listed for the day prior to the discharge of 1 100 000 cu ft per sec at the Carrollton gauge station. Table 13 shows the result of the gauge study at Baton Rouge, together with the studies at Red River Landing and Carrollton. It demonstrates clearly, the increase in gauge for discharging the same volume of water at three points on the Mississippi River below the diversion through Old River.

TABLE 13.—COMPARATIVE GAUGE STUDIES

Location	Period	Years	Average gauge, in feet, for a discharge of 1 100 000 cu ft per sec.	Gauge increase, in feet
Red River Landing	1	1882–1885	41.3	...
	2	1890–1909	45.0	3.7
	3	1913–1932	48.2	3.2
Total	.....	....	....	6.9
Baton Rouge	1	1883–1891	34.0	...
	2	1892–1912	36.0	2.0
	3	1913–1932	38.3	2.3
Total	.....	....	....	4.3
Carrollton (New Orleans)	1	1883–1891	15.0	...
	2	1892–1912	16.6	1.6
	3	1913–1932	17.5	0.9
Total	.....	....	....	2.5

Prior to the flood of 1922 the largest reliable discharge recorded as passing New Orleans was 1 351 000 cu ft per sec, in 1897, at a gauge of 18.6 ft. The flood of 1922 produced a gauge of 21.2 ft for a smaller discharge of 1 281 000 cu ft per sec. During the flood of 1890, a volume of 1 292 000 cu ft per sec passed on the gauge at 16.0 ft, whereas, in 1922, a volume slightly less passed at a gauge of 21.2 ft.

The volume of water diverted into the Atchafalaya is dependent upon the flood height in the Mississippi River. The higher the gauge in the Mississippi at Red River Landing the larger will be the volume of diversion, and the greater the deposition of silt below the point of diversion. The growth or change in the bar below Old River, therefore, is regulated by flood heights and will increase or decrease in extent with the increase or decrease of the average of peak-flood heights. The average peak-flood heights for the period, 1890 to 1907, is 48.7 ft. The flood of 1903, which falls within this group, had a flood peak of 50.1 ft. at Red River Landing, the second highest flood of record to this time. It would be expected, therefore, that the gauge for discharging the same volume of water during this peak-flood year would be reflected by a high gauge.

The locations of diversion as mentioned by Mr. Odom where shoaling in the Mississippi River is not noticeable, are immediately adjacent to the Gulf and below the leveed section of the river, and are not comparable with the effects noted at Red River Landing where the river is confined by levees and subjected to a variation in water elevation of about 50 ft. Baptiste Collette's Bayou is immediately below the end of the East Levee line of the Mississippi;

the Jump at the end of the West Levee; Cubit's Gap and Head of Passes are below the end of the levee system about 7 and 10 miles, respectively. The greater part of the variation in water levels at these points is due to Gulf tides and wind. The river channel has adjusted itself to this condition. On the other hand, if the maximum variation was due to flood heights and if flood heights were increased, causing consequent increase in diversion, shoals would occur. In 1922 the elevation of high water above Mean Gulf Level at these points, were[22]:

Baptiste Collette	4.3
The Jump	4.1
Cubit's Gap	2.0
Head of Passes	2.0

The data presented for the Atchafalaya, which are recorded by the river itself in substantiation of the hydraulic theory, are that since 1890 the same volume of water has been discharging at the same gauge heights as it was in 1933, although the leveed section of the river, including about 17 miles beyond, shows large increases in cross-sectional area. This is further substantiated by the fact that the discharges are the same for the period, 1897 to 1932, for a high gauge of 50 ft at Red River Landing. The velocity during this period shows a slight decrease. Mr. Odom considers that these data are practically no proof and substitutes therefor his opinion, that, when the stream is permitted to discharge freely into Grand Lake by the extension of the levees and the cutting of a pilot channel between them, the full advantage of the increased cross-section will be available. Evidently, he does not consider the inefficient channels below the end of the levee system as being due to the silt load deposited by the Atchafalaya River, and that, after this immediate section of hydraulically inefficient channels is eliminated by the work he suggests, no more will be formed beyond the levee extensions as proposed. If the river's power of adjustment in the handling of its silt load is changed by some method of disposal foreign to the river's treatment, it may create an entirely different condition.

The Atchafalaya is similar to other silt-bearing streams in having numerous hydraulically inefficient channels beyond the end of the levee system. With the drop in velocity, silt is deposited at the levee end, filling in and building up the area below. Due to the presence of colloids, this freshly deposited silt, consolidates and hardens, and requires higher velocities to dislodge it again than it did to keep it moving. This process of land building below the levee end is the foundation utilized by the Louisiana State Board of Engineers in the levee extensions on the Atchafalaya. The levees have been extended in progression with the forming of this foundation by the deposition of silt. The confinement of waters between the extended levees produces sufficient velocity to pick up the consolidated clays again, thereby materially increasing the cross-sectional area of the stream, and transferring the land-building process down stream. If the past is any criterion, that which is now the portion of Grand Lake will, under Mr. Odom's plan, be the location of the numerous hydraulic-

[22] Data furnished by Louisiana State Board of Engrs.

ally inefficient shallow branches when the levees have been extended a sufficient distance and the problem of obtaining a single hydraulically efficient channel will still be present.

The greatest increases in cross-sectional areas of this river are in the lower reaches and are due to levee confinement, as stated by Mr. Odom. In the upper 28 miles of the leveed portion (Reach 0 to 5) the cross-sectional area in 1932 ranged from 70 000 to 79 000 sq ft, being an increase in area from 26 to 35% over that in 1904. In the next following 25 miles (Reach 6 to 10) the cross-sectional area in 1932 ranged from 55 000 to 59 000 sq ft, being an increase in area over that in 1904 from 4 to 208 per cent. The lower 16 miles (Reach 11 to 13) is below the levee system and, in 1932, the area for Reach 11 was 45 000 sq ft, diminishing to 24 000 sq ft for Reach 13, which indicates an area increase over 1904 of 201% and 56%, respectively. Cross-sectional areas are not available prior to 1904–05. Greater increases in area, than those listed, would occur for the period, 1890 to 1932, the period of gauge study. By 1896, the levee system ended within the limits of Reach 6 and was extended into the limits of Reach 8 prior to the cross-sectional survey of 1904–05. Because of this levee extension (which was made prior to the survey of 1904–05), Reach 6 would show a considerably larger increase in area for the prior period over that of 4% for the period since the survey of 1904–05. Therefore, it is logical to consider that the cross-sectional area of Reach 6 was greatly increased during the time of this gauge study (1890–1932) over that of the 4% recorded since 1904–05. This small increase of area for Reach 6, as pointed out by Mr. Odom, is not the total increase in cross-sectional area for the period of this study, and, therefore, does not disturb the conclusions reached as to the adjustment of the river.

It is not the writer's intention to deal with causes of the formation of the present channel of the Mississippi River, but the theory advanced by Mr. Odom on this feature is interesting. The paper deals with the condition of the existing rivers and offers the reason why the Mississippi River is blocked from diverting its flow from its present channel through the Atchafalaya River even if the Atchafalaya is the shortest distance with steeper slopes to the Gulf.

Although the views expressed by Mr. Odom are appreciated, it is thought he did not produce data and conclusions warranting a change in the original deductions.

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DISCUSSIONS

TRUSS DEFLECTIONS: THE PANEL DEFLECTION METHOD

Discussion

By Louis H. Shoemaker, M. Am. Soc. C. E.

Louis H. Shoemaker,[28] M. Am. Soc. C. E. (by letter).[28a]—A number of interesting suggestions have been made by discussers of this paper. Several have proposed, and explained in some detail, methods of calculation in which the effect of the deformation of each truss member is treated separately. Others have proposed special ways of treating certain problems which they appear to consider as offering difficulties if a solution is sought by the method under discussion. These methods, as well as the Williot diagram and the method of "elastic moments," are claimed to offer material advantages, especially as regards the work of computation and avoidance of the use of formulas.

The question of the comparative amount of work required by different methods of computation can only be decided by experience; and in this case there is no doubt as to the result. Formulas, as the name implies, provide a form for the computations to take and, consequently, expedite the work and contribute to its accuracy; and, although these formulas will not be memorized, the manner in which they are developed and applied is easily remembered. Other methods, although not requiring the use of formulas, are much more complicated in their application.

The "panel deflection method" treats the deflections of a truss in exactly the way in which they occur. There is no question but that the panel is the unit of deflection and the equations express the panel deflections in the simplest form.

It is evident that the typical form of panel used applies to all single braced panels with parallel sides, the deflections being taken in the direction of the sides. This type is used in practically all engineering structures.

Note.—The paper by Louis H. Shoemaker, M. Am. Soc. C. E., was published in November, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1936, by Messrs. David B. Hall, and E. Mirabelli; February, 1936, by Messrs. William Bertwell, and Robert H. Hurlbutt; March, 1936, by Messrs. A. A. Eremin, T. P. Noe, Jr., David A. Molitor, and Glenn L. Enke; April, 1936, by Fang-Yin Tsai, Assoc. M. Am. Soc. C. E.; and May, 1936, by Messrs. A. W. Fischer, and L. E. Grinter.

[28] Phœnix, Ariz.

[28a] Received by the Secretary June 22, 1936.

In the case of sub-divided panels, the deflections of the main panel points are obtained from the deformations of the members of the main panel and the deflections of the sub-panel points from the deformations of the members of the triangle forming the sub-panel construction. In trusses having web systems composed of diagonals only, temporary verticals can be considered as applied at the panel points. As these verticals have no stress, the terms in the formulas involving their changes in length disappear. Formulas for the *K*-type of panel are easily developed by the method described in the paper.

Mr. Noe has made an excellent contribution to this discussion by developing the deflection formulas by the use of Williot equations. By applying the formulas to the deflections of an unsymmetrically loaded truss, he has answered the objection that has been raised, that the procedure in this case would necessarily be very complicated.

Mr. Molitor advances the statement, without offering supporting facts, that the method under discussion applies only to very simple truss forms. He refers to "odd-shaped triangular truss systems without vertical members" and complications "due to the more involved geometric relations, for which the new method becomes quite 'out of bounds'." Other than to disagree completely with Mr. Molitor's conclusions in so far as they are definite, it seems unnecessary to reply. However, it is confidently expected that thoughtful and discerning members, who study the facts as published, will conclude that the method offered has the advantages claimed for it.

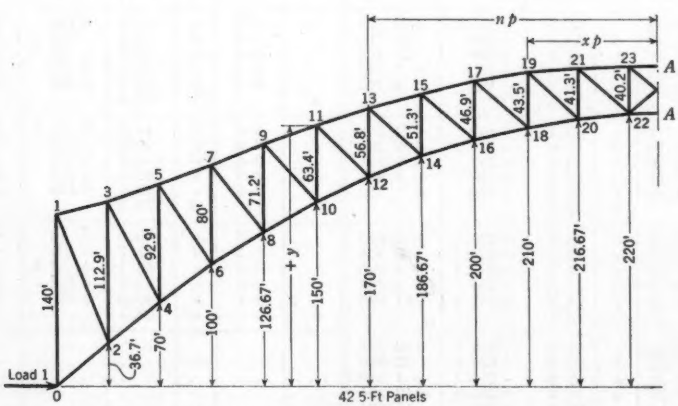

FIG. 23.—ELEVATION OF HALF ARCH, OVER HELL GATE, NEW YORK, N. Y.

An analysis of the steel arch bridge over Hell Gate, in New York City, (see Fig. 23) will serve to answer various criticisms offered in discussion. The computations for the reactions are given in Tables 14 and 15. The preliminary data (that is, changes of length and length of members) were taken from a paper[27] by O. H. Ammann, M. Am. Soc. C. E., published in

[27] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 852.

1918. Table 14 gives the results of the computations of the products $c' \ c$, $d' \ d$, etc., which are the terms composing the formulas. In Table 15 the final results are given. In computing the total deflection of a point Equation (4) gives the total deflection of each point from the origin. Another method of making these computations is to compute the deflection of each point with reference to the preceding point.

TABLE 14.—COMPUTATIONS FOR $c' \ c$, $d' \ d$, ETC., HELL GATE ARCH BRIDGE, NEW YORK, N. Y.

Deflection points (1)	Members (2)	Symbol (3)	Length, in feet (4)	Change in length, in feet × 1 000 *E* (5)	Products, Column (4) × Column (5) (6)	Deflection points (1)	Members (2)	Symbol (3)	Length in feet (4)	Change in length, in feet × 1 000 *E* (5)	Products, Column 4, × Column (5) (6)
23	23-A	*c*	21.25	+347	....		.....	..	.....	....	....
	22-A	*e*	21.25	−150.2	....		.....	..	.....	....	....
22	22–23	*v*	40.2	−120.4	....		.....	..	.....	....	....
21	21–23	*c*	42.6	+708.6	+30 186	9	9–11	*c*	45.3	+296.3	+13 422
	21–22	*d*	57.0	− 88.7	− 5 056		9–10	*d*	64.0	−434	−27 776
	20-22	*e*	42.6	−287.3	−12 240		8–10	*e*	48.5	−132.2	− 6 412
	22–23	*u*	40.2	−120.4	− 4 575		10–11	*u*	63.4	+360	+17 244
	20–21	*v*	41.33	− 19.9	— 756		8–9	*v*	71.2	+405	+19 400
20	.....	*w*	38.0	....	....	8	.....	*w*	47.9	....	....
19	19–21	*c*	42.7	+585	+24 980	7	7–9	*c*	46.0	+270.4	+12 438
	19–20	*d*	56.3	−159.2	− 8 963		7–8	*d*	68.2	−447	−30 485
	18–20	*e*	43.0	−268	−11 524		6–8	*e*	50.2	− 99.7	− 5 005
	20–21	*u*	41.3	− 19.9	− 733		8–9	*u*	71.2	+405	+21 586
	18–19	*v*	43.5	+ 53.3	+ 1 961		6–7	*v*	80.0	+482	+25 690
18	.....	*w*	36.83	....	....	6	.....	*w*	53.3	....	....
17	17–19	*c*	43.0	+545.6	+23 461	5	5–7	*c*	45.8	+196	+ 8 977
	17–18	*d*	56.3	−244.4	−13 760		5–6	*d*	76.0	−334	−25 384
	16–18	*e*	43.7	−239	−10 444		4–6	*e*	52.0	− 88.3	− 4 591
	18–19	*u*	43.5	+53.3	+ 1 967		6–7	*u*	80.0	+482	+30 318
	16–17	*v*	46.9	+128	+ 4 723		4–5	*v*	92.9	+396	+24 908
16	.....	*w*	36.9	....	....	4	.....	*w*	62.9	....	....
15	15–17	*c*	43.4	+492	+21 353	3	3-5	*c*	44.5	+111.6	+ 4 966
	15–16	*d*	57.0	−284.3	−16 205		3–4	*d*	90.2	−359	−32 380
	14–16	*e*	44.5	—207	− 9 211		2–4	*e*	54.0	− 71	− 3 830
	16–17	*u*	46.9	+128	+ 4 864		4–5	*u*	92.9	+396	+31 520
	14–15	*v*	51.3	+204	+ 7 752		2–3	*v*	112.9	+428	+34 070
14	.....	*w*	38.0	....	....	2	.....	*w*	79.6	....	....
13	13–15	*c*	43.9	+429.5	+18 855	1	1–3	*c*	43.6	+ 46	+ 2 005
	13–14	*d*	58.5	−401.1	−23 464		1–2	*d*	111.7	−405	−45 240
	12–14	*e*	45.7	−181.3	− 8 285		0–2	*e*	56.1	+ 53.6	+ 3 010
	14–15	*u*	51.3	+204	+ 8 180		2–3	*u*	112.9	+428	+44 210
	12–13	*v*	56.8	+281.2	+11 276		0–1	*v*	140.0	+387	+39 980
12	.....	*w*	40.1	....	....	0	.....	*w*	103.3	....	....
11	11–13	*c*	44.6	+363.1	+16 194	....	.....	..	.....	....	....
	11–12	*d*	60.8	−420.4	−25 560	....	.....	..	.....	....	....
	10–12	*e*	47.0	−156.2	− 7 340	....	.....	..	.....	....	....
	12–13	*u*	56.8	+281.2	+12 204	....	.....	..	.....	....	....
	10–11	*v*	63.4	+360	+15 624	....	.....	..	.....	....	....
10	.....	*w*	43.4	....	....	....	.....	..	.....	....	....

Professor Tsai has developed equations for the deflections of an irregularly shaped panel by Williot diagrams. The writer appreciates his interesting contribution to this discussion. In developing the "panel deflection method," the writer had in mind its application to engineering structures, and derived the necessary formulas for that purpose in the most direct manner. A complete explanation of the data in Table 1 was included in Example 1

TABLE 15.—FINAL COMPUTATIONS FOR REACTIONS OF HELL GATE ARCH BRIDGE, NEW YORK, N. Y.

Deflection points (1)	Vertical deflection, Δ, in feet, and angular distortion R (2)	Values of $\Sigma\ (Rp)$* and $\Delta + \Sigma\ (Rp)$ (3)	Horizontal deflection, H (4)	Values of y and Ry (5)	Total deflections, Δ_T (6)	Δ_T transposed (7)	Reactions (8)
23	0	....	+347	+260.2	0	54 915	$\frac{55\ 035}{65\ 400} = 0.8417$
22	....	....	....	....	−120	55 035	
	+12.37	....	....	+3 218	....	......	
21	+763	+ 526	+664	+ 258	+ 1 289	......	$\frac{53\ 646}{65\ 400} = 0.820$
20	....	....	....	....	+ 1 269	53 646	
	+ 24.9	+1 289	....	+6 424	....	......	
19	+804	+1 584	+502	+253.5	+ 3 677	......	$\frac{51\ 185}{65\ 400} = 0.783$
18	....	....	....	....	+ 3 730	51 185	
	+ 21.0	+2 388	....	+ 5 324	....	......	
17	+901	+2 477	+412	+246.9	+ 7 054	......	$\frac{47\ 733}{65\ 400} = 0.730$
16	....	....	....	....	+ 7 182	47 733	
	+ 19.0	+3 377	....	+4 691	....	......	
15	+905	+3 284	+314	+ 238	+11 243	......	$\frac{43\ 468}{65\ 400} = 0.665$
14	....	....	....	....	+11 447	43 468	
	+ 16.3	+4 189	....	+3 879	....	......	
13	+984	+3 978	+184	+226.8	+16 205	......	$\frac{38\ 425}{65\ 400} = 0.5876$
12	....	....	....	....	+16 486	38 429	
	+ 14	+4 962	....	+3 175	....	......	
11	+950	+4 572	+ 81	+213.4	+21 727	......	$\frac{32\ 828}{65\ 400} = 0.502$
10	....	....	....	....	+22 087	32 828	
	+ 11.6	+5 522	....	+2 475	....	......	
9	+922	+5 065	− 20	+197.9	+27 714	......	$\frac{26\ 796}{65\ 400} = 0.4096$
8	....	....	....	....	+28 119	26 796	
	+ 9.3	+5 987	....	+1 840	....	......	
7	+906	+5 460	− 89	+180	+34 080	......	$\frac{20\ 352}{65\ 400} = 0.311$
6	....	....	....	....	+34 562	20 352	
	+ 8.1	+6 366	....	+1 458	....	......	
5	+809	+5 804	−114	+162.9	+40 693	......	$\frac{13\ 825}{65\ 400} = 0.2112$
4	....	....	....	....	+41 089	13 825	Correction 0.0015
	+ 5.4	+6 613	....	+880	....	......	0.2127
3	+741	+6 034	−116	+149.6	+47 468	......	$\frac{7\ 018}{65\ 400} = 0.1072$
2	....	....	....	....	+47 896	7 018	Correction 0.0032
	+ 5.1	+6 775	....	+763	....	......	0.1104
1	+810	+6 250	−136	+140	+54 528	......	0
0	....	....	....	....	+54 915	0	Correction 0.004
	+ 4.3	+7 060	....	+602	....	......	0.004
			+2 029	+34 729			

By Equation (5), $H_T = 2\ 029 - 34\ 729 = 32\ 700$; $32\ 700 \times 2 = 65\ 400$

* $p = 425$ ft.

of the paper. Professor Tsai is in error in thinking this method does not apply to trusses with sub-divided panels. The writer believes that his explanation, given previously in this discussion, fully answers this question.

Professor Grinter has misinterpreted the description of the method under discussion. The writer's reference to the deflections of a beam was for the purpose of calling attention to similarity of methods. A careful study of

the paper will reveal the fact that deflections are computed with reference to a point called the origin of deflections and a line through this point called the axis of deflections. The deflection of a single point of a structure, furthermore, can be computed as easily by the "panel deflection method" as by the method of virtual work.

The deflection of the nth panel point with reference to the $(n-1)$th panel point is,

$$\Delta_n = \Delta_1 + \sum_{x=0}^{x=n-l} (R\ p) \qquad (47)$$

and,

$$\Delta_T = \Delta_T + \Delta_n \qquad (48)$$

This method of computing Δ_T enables the results to be tabulated in more compact form. In Table 15, Δ and R are given together in Column (2), Δ on the upper line and R on the lower line of the panel space. In the same manner $\Sigma\ (R\ p)$ and $\Delta + \Sigma\ (R\ p)$ are given in Column (3) and y and $R\ y$ in Column (5). In obtaining the values of Δ_T, the value of $\Delta + \Sigma\ (R\ p)$ on the bottom line of a given panel space is added to the value of Δ_T in the panel space above, and the result is the value of Δ_T for the given panel. The computations for horizontal deflections of Point O are written at the bottom of the table. It is believed that a comparison of Tables 14 and 15 with the tables of computations, made by Mohr's method of elastic moments,[27] is a sufficient answer to the various claims and criticisms that have appeared in this discussion.

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DISCUSSIONS

LATERAL PILE-LOADING TESTS

Discussion

By D. P. Krynine, M. Am. Soc. C. E.

D. P. Krynine,[17] M. Am. Soc. C. E. (by letter).[17a]—The lateral pile-driving tests at Alton, Ill., described in this paper, were made on a considerably larger scale than any other experiments known to the writer. Therefore, they approach the actual loading conditions more closely. Furthermore, the piles were studied both as single units and in groups, with heads embedded in concrete monoliths, whereas other investigators have dealt with isolated piles only.

Previous Experiments.—There is a series of older German experiments made for the purpose of determining the resistance of posts placed in narrow holes which subsequently were back-filled with earth. Among the experiments of this series are those by Professor Engel[18], H. Fröhlich[19], and those by H. Dörr[20]. The latter investigator used posts about 18 ft high, and the depth of embedment varied between 4.3 and 6.1 ft. The soil was Rhine sand, as a rule finer than 1 mm, with the angle of natural slope about 40 degrees. Dörr measured horizontal forces corresponding to horizontal displacements of a certain point on the test post, care being taken to substract therefrom the purely elastic deformation of the post itself. The horizontal forces did not exceed 700 lb; and the piles tested were rectangular, about 6 by 9 in. in cross-section, or round, about 8 in. in diameter. The tests were conducted to the point at which the soil mass failed so that the ground close to the pile cracked and an opening was formed behind it. This hole was filled with fine ashes and the post was hammered down somewhat. Subsequent excavation showed the absence of ashes in the lowest third of the embedded depth. Thus, the existence of the "zero point" was established experimentally. An isolated pile embedded in the ground and subjected to the action of a horizontal force, would rotate about that zero point.

Note.—The paper by Lawrence B. Feagin, Assoc. M. Am. Soc. C. E., was published in November, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1935, by A. E. Cummings, Assoc. M. Am. Soc. C. E.; January, 1936, by Messrs. J. C. Meem, and T. Kennard Thomson; February, 1936, by August F. Niederhoff, Jun. Am. Soc. C. E.; and August, 1936, by Messrs. Lazarus White, and Y. L. Chang.

[17] Research Associate in Soil Mechanics, School of Engineering, Yale Univ., New Haven, Conn.

[17a] Received by the Secretary July 8, 1936.

[18] *Zentralblatt der Bauverwaltung*, 1903, p. 274.

[19] "Beitrag zur Berechnung der Mastfundamente", 1913.

[20] "Die Standsicherheit der Masten und Wände im Erdreich", 1922.

Experiments conducted at North Carolina State College, Raleigh, N. C., and by the State Highway Board of Georgia[21] have shown that the resistance of a pole to overturning is proportional to the square of the penetration depth. This fact induces one to believe that the resistance of a pile to the action of a lateral force is connected with the configuration of some body of rotation which develops within the earth mass close to the pile under consideration. The North Carolina tests were conducted in hard clay, and those in Georgia in clay containing about 50 to 60% of sand,

Introducing the following notations: P_0 = an overturning force, in pounds; d = depth of embedment or penetration, in feet; and c = a coefficient. The overturning force may then be expressed as:

$$P_0 = c\,d^2 \qquad (57)$$

The value of the coefficient, c, was found to be 1 250 in North Carolina and 300 in Georgia.

In 1928 the writer made some field and laboratory tests on pile models loaded horizontally. These tests were made in co-operation with Messrs. G. I. Pokrowski and N. V. Laletine[22] who later continued this work alone.[23] Wooden

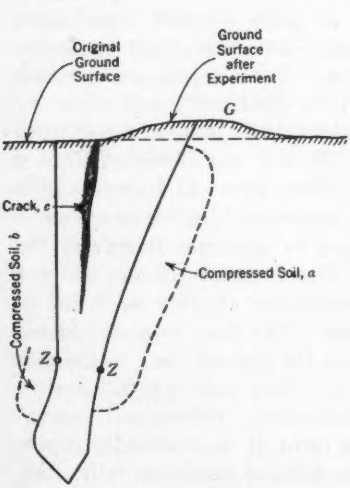

FIG. 29.

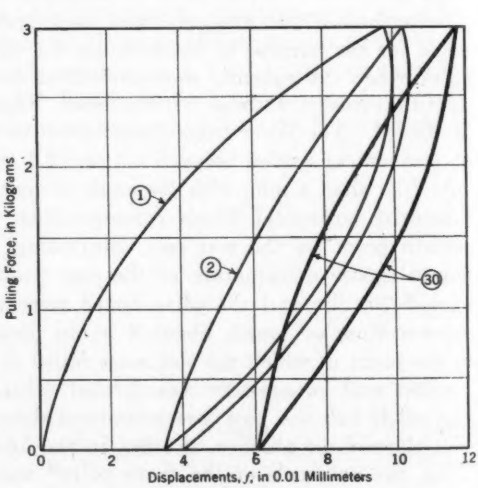

FIG. 30.

piles, 3 to 3½ in. in diameter, ranging in length from 6 to 7½ ft, were driven into a very uniform clay with 17 to 18% of natural moisture. Then the piles were overturned by applying a horizontal force. A cross-section through the center of the pile was excavated carefully, using shovels and finishing the vertical surface with a sharp knife. Fig. 29 is a sketch made from a photograph of that cross-section. The zero point, Z, located at a depth from

[21] *Wood Preservation News*, November, 1932, Vol. 10, pp. 156–158.
[22] "Soil Investigations in 1928–1929" (printed in Russian), pp. 5–21, 65–67.
[23] "Soil Investigations in 1929–1930" (printed in Russian), pp. 15–33.

0.52 d to 0.69 d, is clearly seen; and it appears that the overturning process consists of two phases. In the first phase the pile rotates slightly about its zero point, Z, being resisted by both the compressive resistance of the soil (compressed zones, a and b) and by its tensile strength (Crack c). The second phase is characterized by the breaking down of the shear resistance. The center of rotation gradually moves from the zero point, Z, to the surface of the ground, G, following the curve, ZG; and, finally, an earth cone is pushed out. A weak pile breaks during this second phase.

It follows from these experiments that a structure founded on piles and subject to lateral forces, is kept in place by the compressive resistance of the adjacent earth mass. This compressive resistance, however, does not control the ultimate, or the limit of equilibrium of the structure. As soon as the acting force overcomes the value of the shearing resistance of the soil, the deformation takes the path of least resistance, and the mass fails by shear. Hence, the actual lateral force should not be greater than an nth part of the shearing resistance, in which n is the safety factor. Furthermore, there is a sharp difference between the failure of the earth mass and the failure of the structure itself, since owing to the excessive compressibility of an earth mass, the structure may move laterally and fail before the adjacent earth mass is disturbed by shear. A further development of these experiments has been discussed by the writer elsewhere.[24]

There are two more series of experiments of this kind. In 1928 and 1930, Paul E. Raes, of the University of Ghent, Belgium, made some tests using concrete piles, 12 by 12 in. and 14 by 14 in. He published his results and advanced a theory which is essentially that of a zero point, or "pivot" according to his terminology[25]. In 1933, M. Nakamura[26] made some very precise small-scale experiments concerning lateral pile action. He used wooden and metallic pile models (from $2\frac{5}{16}$ to $3\frac{5}{16}$ in. in diameter) embedded about 10 to 12 in. in a sand mass confined in a box, 24 by 24 in. by 16 in., which was supposed to be large enough to leave the model pile action undisturbed. A horizontal force up to 10 lb was applied at levels of about 22 to 28 in. above the surface of the soil in the box. Deformations of the embedded part of the pile were measured, and the zero point was located at about seven-tenths the depth of embedment. Fig. 30 represents the horizontal displacements of the pile at the level of the ground under the repeated action of a force gradually increasing from zero to about 6.6 lb and decreasing to zero again. Displacements consisted of both an irreversible and an elastic part; but after a certain number of loading and unloading cycles there was elastic action only. (Note the heavy lines in Fig. 30.)

Lateral Pile Action and Loading Tests Interrelated.—Mr. Feagin states that "frequent repetition of lateral loads results in slightly greater lateral movement than a sustained load of the same magnitude." This is because a repetition of load application tends to eliminate gradually irreversible lateral

[24] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), pp. 271–273.

[25] *Proceedings*, International Conference on Soil Mechanics (1936), Vol. 1, Paper H-1; also, *La Technique des Travaux*, No. 8, p. 505 (1928).

[26] "Ueber den Erdwiderstand der Maste", *Der Bauingenieur*, Vol. 16, pp. 269–275, 1935.

movements, as shown in Fig. 30. The case of a pile acted upon by a horizontal force is a special case of the loading of an earth mass; hence analogies should be sought between such an action and the action of a loaded plate placed at the surface of an earth mass. Fig. 31 shows the settlement curve in an experiment made by the writer[27]. A confined sand layer, 3 in. in diameter and about $1\frac{1}{4}$ in. thick, was subjected to compression, using loads ranging from 16 to 65 lb per sq in. applied to a metallic piston acting at the sand surface. Curves A and B, Fig. 31, correspond to the total settlement of the sample in the loaded and unloaded condition, respectively, so that the difference between them denotes the elastic rebound at a given stage of the experiment. Similar curves may be obtained by joining the upper and the lower points of the curves in Fig. 4(e), or in Fig. 9.

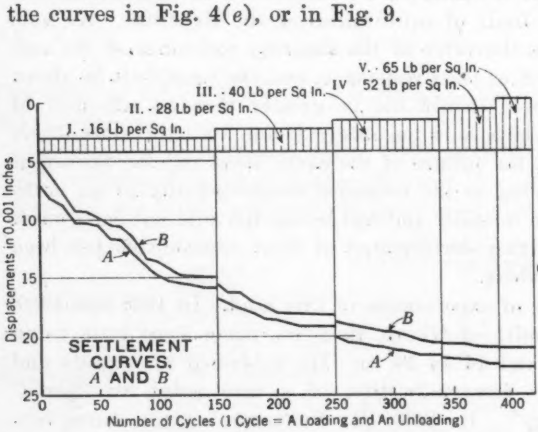

FIG. 31.

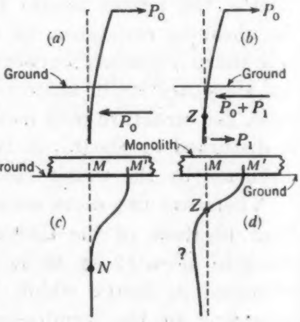

FIG. 32.

In connection with Fig. 30 it should be stated that each pair of curves corresponding to a loading and unloading cycle, is analogous to a hysteresis loop in loading experiments. Order numbers of loading and unloading cycles are indicated by the numbers in circles in Fig. 30. After a certain number of such cycles (thirty in Fig. 30) the hysteresis loops become closed, which means that the earth mass has attained perfect elasticity under the action of a given stress. This elasticity is not the same, however, as that required by Hooke's law, since displacements are not proportional to the stresses. It is also remarkable that this state of elasticity may be destroyed as soon as a stress greater than that used in compression is applied (see Fig. 31).

"Pre-Testing" of Piles by Lateral Pull.—If instead of a confined sand sample (Fig. 31) a plate is placed at the ground surface and loaded repeatedly, the results would be practically the same[28]. If a pile is subjected to repeated vertical loading, an analogous action occurs. Using this property, Lazarus White, M. Am. Soc. C. E., developed a so-called "pre-test" method of foundation construction applied principally to piles[29]. It is well known that the

[27] *Proceedings*, International Conference on Soil Mechanics (1936), Vol. II, pp. 22-23.

[28] "Underpinning", by E. A. Prentis and Lazarus White, M. Am. Soc. C. E., p. 241, Fig. 128 (1931).

[29] *Loc. cit.*, p. 113, Fig. 61.

surface of an earth road that is reasonably moist becomes elastic under traffic; and this surface is easily destroyed should a few heavy loads pass over the road. All kinds of pavements are automatically "pre-tested" by traffic. Assembling all the foregoing facts, it may be stated that practically every earth structure, except perhaps the case of saturated plastic soils, can be "pre-tested" within the meaning of the term introduced by Mr. White. This suggests that irreversible deflections of piles subjected to lateral forces could be also eliminated by duly "pre-testing" them during construction. How this "pre-testing" could be done—pile by pile, or otherwise—and whether such a method is economically feasible are questions to be decided for each individual case.

Theory of the Zero Point.—Fig. 32 represents a hypothetical weightless pile loaded at its top with a horizontal force, P_0. If there is no zero point, the pile would deflect as shown in Fig. 32(*a*). The resultant of earth reaction, P_1, would form an unbalanced couple with the acting force, P_0, and this is incompatible with conditions of equilibrium. Hence, the existence of a zero point is a mechanical necessity, the acting force, P_0, and the reaction, P_1, of the earth at the bottom part of the pile being balanced by the reaction, $P_0 + P_1$, at the upper part (Fig. 32(*b*)). A most comprehensive mathematical theory concerning the location of the zero point has been advanced by K. Hayaschi[30] in describing the behavior of a beam on a continuous elastic support. The Russian Professor I. P. Prokofiev has advanced a simplified theory of the zero point[31]. The existence of the zero point was also proved mathematically by Mr. Prudon[32]. The zero-point theory has been discussed by C. C. Williams[33], M. Am. Soc. C. E. Mr. Raes[25] advanced a mathematical theory of the zero point ("pivot"), including the case of two piles with heads embedded in a concrete monolith. Mr. Raes comes to the conclusion that in the latter case one of the two piles is pulled, and the other pushed, and that the "pivot" exists in all cases.

In the experiments by Mr. Feagin there is no direct indication as to whether or not there is a "pivot" in the case of piles with embedded heads. The elastic line in this case may be visualized either according to Fig. 32(*c*) or Fig. 32(*d*). In the former case there are vertical tangents at Points *N* and *M′*, and the elastic line in the latter case has been drawn quite arbitrarily only to show that, in the opinion of the writer, there is a zero point located rather high. Mr. Feagin testifies that between the pile and the concrete (Point *M′*) there was "a crack in which the blade of a pocket knife could be inserted". Hence, it is doubtful whether the tangent to the elastic line at Point *M′* was vertical. On the other hand, in Fig. 32(*c*), the line, *NM′*, is longer than the line, *NM*; and neglecting the deformations of the pile, one may conclude that the pile should be pulled out at Point *N*, which contradicts the assumption of ideal embedment at that point. The writer believes that only in the case of feebly rigid piles the assumption of Fig. 32(*c*) may

[30] "Theorie des Balkens auf elastischer Unterlage", Berlin, 1921, p. 228 *et seq.*

[31] "Theory of Structures", Pt. II, p. 232 *et seq* (printed in Russian in 1928).

[32] *Le Génie Civil*, No. 5, 1926, p. 117.

[33] "The Design of Masonry Structures and Foundations", Second Edition, 1930, p. 478 *et seq.*

lead to a rather satisfactory working theory. In a general case, however, the attention of the investigators should be directed toward the location of the zero point, Z (Fig. 32(d)). Mr. Feagin states: "It was found that a carpenter's rule could be inserted to a depth of 6 ft in the space between the pile and the adjacent foundation sand on the side of the hole nearest the jack." Probably that rule reached Point Z, as shown in Fig. 32(d).

Extension of the Experiments Suggested.—The excellent experiments by Mr. Feagin should be extended somewhat further: First, it seems necessary to investigate whether or not the "pre-test" method as applied to piles deflected laterally can decrease the lateral movement of locks; and, second, the shape of the elastic line of such piles should be investigated more closely. For economy, smaller pile models driven into uniform natural clay may be used in these particular field tests. It would be very important, furthermore, to study the characteristics of batter piles.

Additional Remarks on Irreversible Earth Deformations.—The following observation by Mr. Feagin is to be emphasized: A pile loaded horizontally never returns to its original position upon removal of the load. Actually, even a perfectly elastic pile is restrained from returning to its initial position owing to irreversible deformations of the earth mass. This fact may cause serious secondary stresses in a framework, such as a wooden bridge.

Action of Forces at a Lock.—If there were no piles, horizontal stresses due to the difference of levels in both the upper and the lower pool (Fig. 3(a)) would develop under the structure. The piles driven into the ground form, with the soil, a mass of "reinforced earth", and it is very probable that the horizontal stresses referred to, are taken up by that mass which works as a whole. A graphic method of computing these horizontal stresses has been advanced by the writer elsewhere[34]. Assuming the earth material under the structure to be elastically isotropic, the value of such a stress pushing the structure down stream, would be approximately: $0.5 \times 62.5 \times (38.0 - 15.2) = 713$ lb per ft of width of the structure and per lin ft of the length of the piles. This value should be less in the case of sands. The values given do not pretend to any degree of accuracy; the writer wishes only to state that an additional force pushing the lock down stream may exist and that this force is due to the difference of water levels in the pools.

Conclusion.—As a rule, the closer a soil model is to a full-sized structure, the more trustworthy are the results. In this respect Mr. Feagin's experiments with large models are unusually interesting. To increase the value of the experiments, it seems necessary to extend their scope somewhat, but even in their present form, Mr. Feagin's experiments are an important contribution to the field of soil mechanics and a valuable source of information for both the practical engineer and the theoretical worker in that field.

[34] *Proceedings*, Am. Soc. C. E., April, 1936, p. 532, Fig. 19.

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DISCUSSIONS

TALL BUILDING FRAMES STUDIED BY MEANS OF MECHANICAL MODELS

Discussion

By MESSRS. GEORGE E. LARGE AND SAMUEL T. CARPENTER, AND A. W. FISCHER

GEORGE E. LARGE,[14] ASSOC. M. AM. SOC. C. E., AND SAMUEL T. CARPENTER,[15] JUN. AM. SOC. C. E. (by letter).[15a]—Much painstaking work is represented in the long series of tests on wind-bent models described in this paper. Although the writers cannot agree that all the conclusions of the study have general application to modern wind-bent design, they believe that this paper may be the means of defining the fields of at least two well-known methods of design with which they have experimented since 1933.

The Spurr adaptation[16] of the Cross moment distribution method has been used, as in Fig. 21, to verify the authors' experimental results. The regularity of the model framing will throw the contraflexure points at mid-story of the columns, as assumed in checking, except in a few of the lower stories where, because of the fixed column bases, the points of contraflexure will be above the half-story level, and, in some cases, absent entirely. Nevertheless, the ratio of girder shears across a floor will be substantially the same throughout all stories, as will be the ratio of column reactions, R_i, which is the crux of the argument.

To compensate for the vertical displacement of girder ends due to floor-joint rotation, Mr. Spurr uses a reduced length for girder bending equal to $\sqrt{\frac{F^3}{F+D}}$ in which F is the distance face to face of columns, and D is the

NOTE.—The paper by Francis P. Witmer, M. Am. Soc. C. E., and Harry H. Bonner, Esq., was published in January, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1936, by Messrs. L. C. Maugh, L. J. Mensch, and Gilbert Morrison; and August, 1936, by Fang-Yin Tsai, Assoc. M. Am. Soc. C. E.

[14] Research Assoc. Prof., Eng. Experiment Station, The Ohio State Univ., Columbus, Ohio.

[15] Instructor in Civ. Eng., Swarthmore Coll., Swarthmore, Pa.

[15a] Received by the Secretary July 23, 1936.

[16] "Wind Bracing of Steel Buildings," Second Progress Rept. of Sub-Committee No. 31, Committee on Steel of the Structural Division, Discussion by H. V. Spurr, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., August, 1932, p. 1133, and Fig. 26.

depth of column section. For a 4-in. model span it is $\sqrt{\frac{(3.5)^3}{3.5 + 0.5}} = 3.28$ in.[17]

In computing the stiffness, $\frac{I}{L}$, of the members, the center-to-center lengths of columns and the foregoing "Spurr lengths" of girders were utilized, since their use has theoretical foundation and, ordinarily, gave results nearer to experimental values both on the authors' models and on those studied by the writers.

Columns (2) and (3), Table 3, show a comparison of the principal experimental and calculated results. The upper group of three cases check very

TABLE 3.—COMPARISON OF EXPERIMENTAL AND COMPUTED RESULTS

Case	R_1, INNER COLUMN REACTION FOUND, IN PERCENTAGE OF OUTER COLUMNS.			CUSTOMARY ASSUMPTIONS AS TO COLUMN REACTION (R_1)		$\frac{\text{Column}}{\text{Girder}}$-stiffness, ratio $\frac{\Sigma\text{ (columns)}}{\Sigma\text{ (girders)}}$
	Experimental result	Calculated as a check by moment distribution	Remarks	(Portal)	(Cantilever)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
A-1	−15	−14	"O.K"	0	+33	1.75
C-2	0	+0.3	Equals portal performance	0	+25	1.05
C-0.5*	+28	+27.0	Integral connections, 55 stories.	0	Designed +28	Lower 11.2 story
B-9	+13	+3	Approximately portal performance	0	+50	0.62
B-17	+46	+36	Extreme girder proportions	0	+50	0.39
A-2	+33	+50	Questionable cantilever performance	0	+33	1.31
A-17	+230	+340	Extreme girder proportions	0	+33	0.28
11-*C*-2	None	+20	Approximately cantilever performance	0	+25	11.6
5-*B*-9	None	+62	Approximately cantilever performance	0	+50	3.1

* Model tested by writers.

closely, indicating the correctness of the calculation method as well. Serious discrepancies occur in the next group of four cases. Some extreme cases of panel-to-panel girder proportioning exist in this group, and it seems very likely that the connection blocks were inadequate for holding the very stiff girders then used, causing experimental error. It is the writers' experience that one cannot make a theoretically rigid connection, as assumed in the calculation methods, except by developing tremendous frictional forces or by making an integral connection. Perhaps the authors, thinking of practical wind connections, did not desire complete rigidity. Nevertheless, the connections are variables which obscure the picture of girder performance.

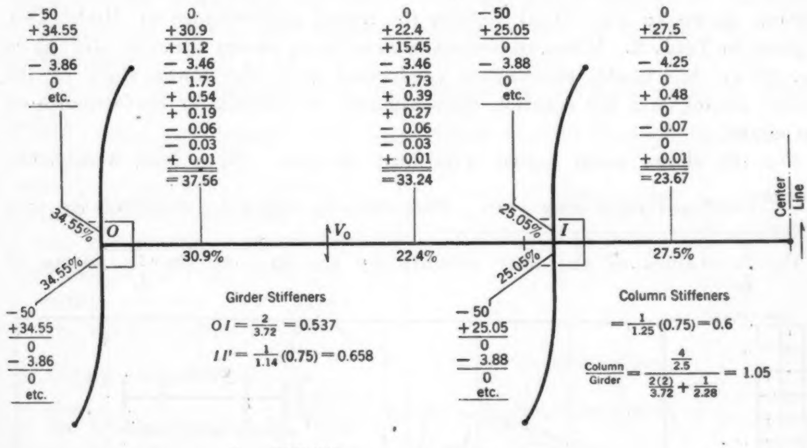
The principal comments which the writers wish to make are as follows:

(*a*) In constructing models of tall building bents, the stiffness of the column system relative to the girder system must be reproduced faithfully,

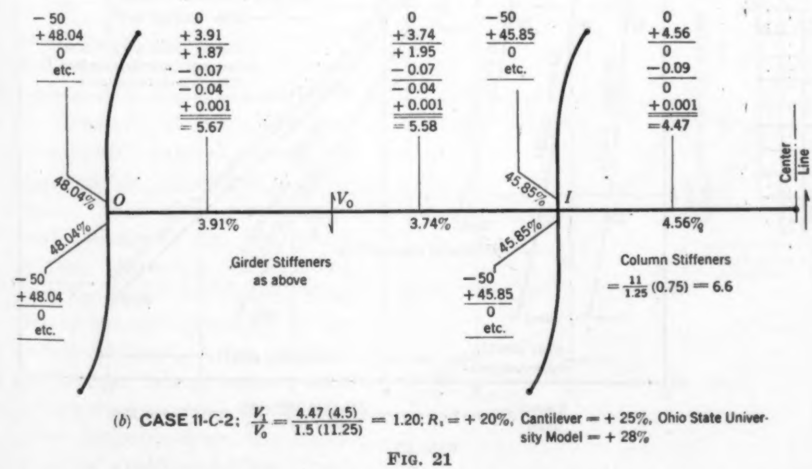
[17] "Tests and Design of Steel Wind Bents for Tall Buildings," by George E. Large, Assoc. M. Am. Soc. C. E., Samuel T. Carpenter, Jun. Am. Soc. C. E. and Clyde T. Morris, M. Am. Soc. C. E., *Bulletin No. 93*, Appendix, Eng. Experiment Station, Ohio State Univ., Columbus, Ohio.

since their ratio has tremendous effect upon the distribution of girder shears and column reactions across a bent.

(*b*) The model columns in the authors' tests were so flexible relative to the model girders that most of the models did not represent practical wind bents. The experimental data, therefore, were not sound bases for drawing general conclusions in regard to the distribution of column reactions.



(*a*) CASE C-2: $\frac{V_1}{V_0} = \frac{23.67\,(4.5)}{1.5\,(70.8)} = 1.003$; $R_1 = +0.3\%$



(*b*) CASE 11-C-2: $\frac{V_1}{V_0} = \frac{4.47\,(4.5)}{1.5\,(11.25)} = 1.20$; $R_1 = +20\%$, Cantilever = + 25%, Ohio State University Model = + 28%

FIG. 21

(*c*) A distinction should be made between wind bents upon the basis of slenderness. Design methods and assumptions which are good enough for 20-story bents can be actually dangerous when applied to a 60-story bent of the same base width.

(*d*) For a fair comparison[18] of the several methods of assigning girder shears and column "chord pulls", with economy in view, one must contemplate designs having the same calculated total lateral deflection, including that due to lengthening and shortening of columns.

(*e*) Girder systems selected for strength only, without regard for stiffness, permit too much flexibility in a modern slender tower.

Fig. 21 is submitted in support of Item (*a*). The moment-distribution solution shown in Fig. 21(*a*) verifies the portal performance of Model *C*–2, as given in Table 3. When the columns were made eleven times as stiff, as in Fig. 21(*b*), the model dimensions conformed with the lower story of the writers' model, and the solution approximated the cantilever performance of that model.

Fig. 22 shows some actual wind-bent designs. Note how widely the $\frac{\text{column}}{\text{girder}}$-stiffness ratio may vary. This ratio is found by dividing the sum of the $\frac{I}{L}$-values of the four columns by the sum of the $\frac{I}{L}$-values of

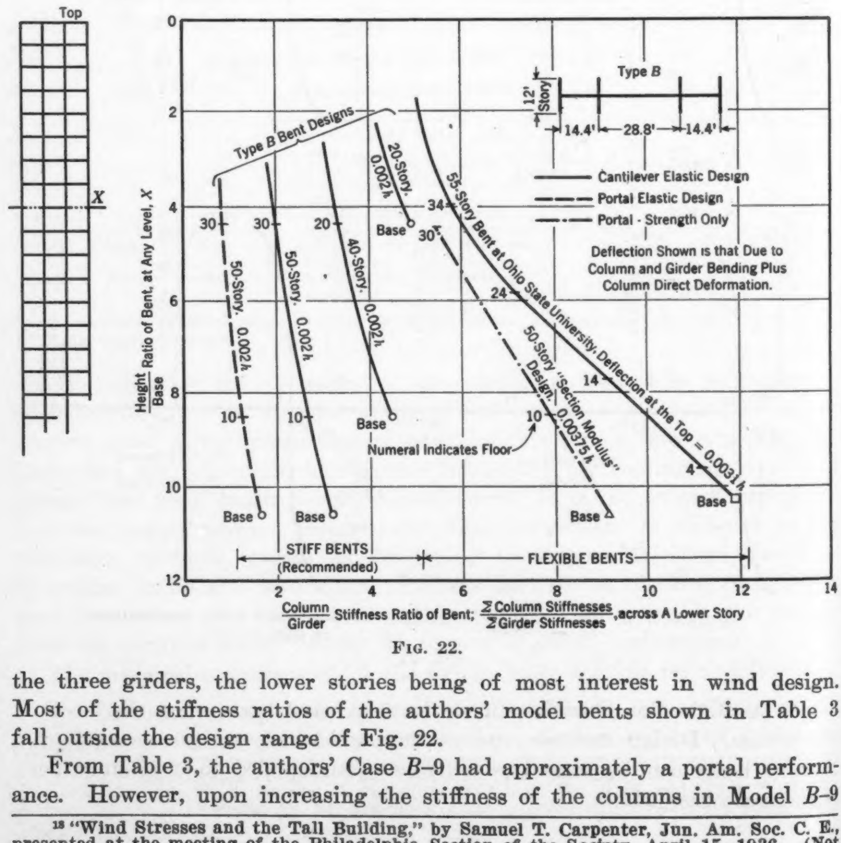


FIG. 22.

the three girders, the lower stories being of most interest in wind design. Most of the stiffness ratios of the authors' model bents shown in Table 3 fall outside the design range of Fig. 22.

From Table 3, the authors' Case *B*–9 had approximately a portal performance. However, upon increasing the stiffness of the columns in Model *B*–9

[18] "Wind Stresses and the Tall Building," by Samuel T. Carpenter, Jun. Am. Soc. C. E., presented at the meeting of the Philadelphia Section of the Society, April 15, 1936. (Not published.)

five times, it conformed closely with the lower story of the 50-story cantilever design, in which the $\frac{\text{column}}{\text{girder}}$ -stiffness ratio is 3.16, and had a cantilever performance. It is designated as Case 5–*B*–9 in Table 3. (The increased size of the inside columns of the practical designs prevents perfect checks.)

Similarly, by increasing the columns of (portal) Case *B*–9 eight times, the cantilever performance of the 20-story cantilever design can be verified approximately. Obviously, column stiffnesses have considerable to do with performance.

Fig. 23 is a summary of the design of several bents of varying $\frac{\text{height}}{\text{base}}$ -ratios. The spans taken were chosen because they are generally considered favorable to the portal method. Both cantilever and portal designs were made for each height. All the bents represented by the two upper curves were designed to have a total deflection at the top of $\frac{1}{500}$ of the height or 0.002 h. The column selections were the same in all bents of the same height, except in the case of the 40-story and the 50-story portal-designed bents, in which extra column area had to be provided on account of the increased "wind-pull", carried entirely by the outside columns in this case. This extra was considered chargeable to the girder system, for fair comparison with the cantilever design. Note the economic advantage of the cantilever method for shallow-connection bents higher than twenty-five stories. It is also most desirable from the standpoint of avoiding column chord deformation accumulation.

These girders were designed as follows: With columns set, the portion of the entire 0.002 h deflection which was due to

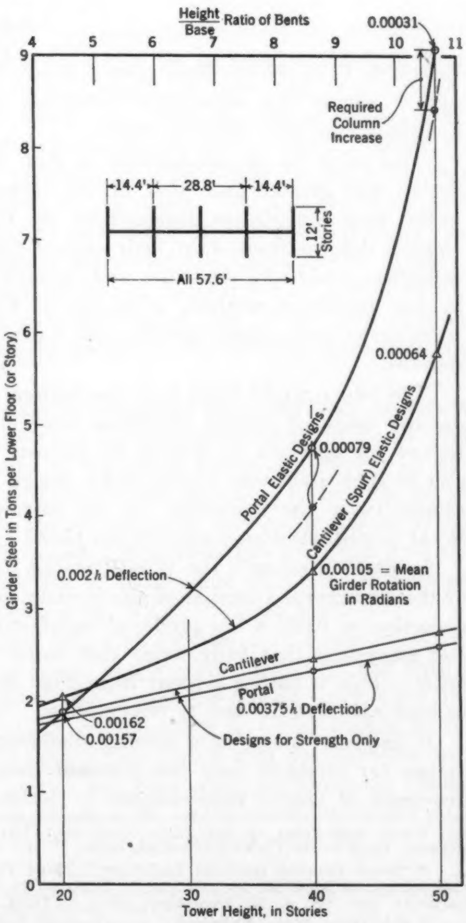


FIG. 23.

chord deformation of columns was calculated quickly by the following formula[19] for chord deflection:

$$y_T = \frac{2\, s_w\, h^2}{3\, E\, B} \quad \text{.......................(39)}$$

in which s_w is the wind unit direct stress in the outside basement column; h is the full height of the bent; E, the elastic modulus; and B, the base width of the bent. The chord deflection was deducted from 0.002 h to find how much deflection was to be contributed by the bending of girders and columns, and girders were selected on the basis of required moment of inertia by the formula:

$$I_G = \frac{V\, b^2}{3\, E\, \theta} \quad \text{.......................(40)}$$

in which V is the assigned girder shear; b is the half-length of the girder; and θ equals the remaining deflection assignable to girders, divided by h, equals the joint rotation angle.

The point to be emphasized is that the chord deflection of the portal design was greater than that of the corresponding cantilever design, which meant that the girders designed by the portal method had to have smaller angular deformations, θ, at their ends if the bents were to have the same total deflection, and had to be stiffer and (usually) heavier than the girders designed by the cantilever method, as shown in Fig. 23. The small numerals near each curve point show in each case the required girder rotation angle, θ, in radians.

The two straight lines near the bottom of the graph show girders selected on the basis of section modulus solely to meet allowable stress, without regard for stiffness as defined by moment of inertia. As seen in Figs. 22 and 23 such a 50-story portal girder design has about one-half the weight, but almost twice the deflection, of the corresponding cantilever elastic design. What priced wind-bracing will you have?

In its 1933 report[20], the Wind-Bracing Sub-Committee, Committee on Steel of the Structural Division of the Society, recognized that limiting the design deflection to 0.002 h has produced satisfactory results[21] in very slender towers, and suggested that fully twice that value may be allowable in less extreme bents. This is rather evident from Fig. 23 if one compares bents of heights to base ratios of 10 and 4, respectively.

It has been found by a moment-distribution check that the 50-story portal design for strength only has reversed interior column reactions due to the disregard of elastic requirements in designing. On this account the chord

[19] For derivation of Equations (39) and (40) see *Bulletin No. 93*, Eng. Experiment Station, Ohio State Univ., Columbus, Ohio.

[20] "Wind Bracing in Steel Buildings," Third Progress Rept. of Sub-Committee No. 31, Committee on Steel of the Structural Division on Wind-Bracing for Steel Buildings, *Proceedings*, Am. Soc. C. E., December, 1933, p. 1614.

[21] The Ohio State University experimental bent was designed to have 50% greater deflection in order to facilitate the detection of joint rotations (θ). (See Fig. 22.)

deflection of this bent had to be calculated by a more general formula than Equation (39), and was found to be 0.00185 h, which is greater than in the portal elastic design where it was 0.00163 h. In order to have the bent under consideration deflect only 0.002 h for fair comparison with the elastic designs, the girder contribution would have to be reduced to about 0.00009 h, since the columns will contribute about 0.00006 h. This would require ridiculously large girders, twenty times as stiff, and at least five times as heavy, as those represented in Fig. 23, exceeding even the portal elastic design in weight.

Allowable values for deflection should be established as in the case of stress in order to insure against the application of 20-story methods to 50-story towers. Deflection data from the authors' models would be enlightening.

A. W. FISCHER,[22] ESQ. (by letter).[22a]—A considerable mass of experimental data as to the values of the vertical reactions of the columns at the first story of three-bay structures with equal vertical heights, is presented in this paper, and the authors' results seem to agree very well with those obtained by mathematical analysis. Additional experiments should be made, however, to determine the ratio of the vertical reactions at other stories (especially at the top story) before a general conclusion can be made. The authors' state that for equal bays (all columns being of the same size), the cantilever relation can be obtained by making the moment of inertia of the center girder approximately twice that of the side girders, or that if the moment of inertia of the center girders was about one and one-third times that of the side girders, zero reaction would result for interior columns and would thus agree with the portal-theory distribution.

The writer applied the slope deflection method to a two-story structure with sizes of girders and columns as given for the authors' Case A-1 and with a horizontal unit load of 1 lb applied at the top of the second story. In this manner it was found that $R_A = + 0.2976$ lb and $R_B = - 0.05065$ lb, which makes R_i for the first story equal to -17%, and this value agrees fairly well with the results reported in the paper. If more stories had been used the results would be changed somewhat, but still the value of R_A would have an opposite sign from R_B. For the second story, $R_i = - 21$ per cent.

The writer also applied the slope deflection method to a two-story structure with the sizes of girders and columns as given for Case A-17 and with a horizontal unit load of 1 lb applied at the top of the second story. For the first story, $R_A = + 0.14985$ lb, and $R_B = + 0.43935$ lb, which makes R_i for the first story equal to + 293 per cent. This result does not agree so well with that of the paper. If more stories had been used, no doubt the results would change somewhat, but still the values of R_A and R_B would be of the same sign, as the authors' experiments show. For the second story, $R_i = + 243$ per cent.

[22] Care, Pennsylvania Sugar Co., Philadelphia, Pa.

[22a] Received by the Secretary August 11, 1936.

With the so-called exact methods of analysis the sizes of the girders and columns must be known before the analysis can be started, especially in the case of the slope deflection method and the Cross method, and, therefore, the methods are rather tedious for the design of a new structure.

In his analysis of a three-span structure (equal spans) by relative deflections, Professor H. Yu[23] assumes that the ratio of the columns to the girders is 1.5, the story height being equal to 1.0 and the span length to $\frac{4}{3}$. For a new design, three bays wide and with equal story heights, Professor Yu's method is very flexible because, with the span and story height given, the ratio of the moment of inertia of the columns to that of the girders (the moment of inertia is the same for the columns in each story, but for different stories it will be different) can be selected so as to fit approximately the relation assumed by Professor Yu in deriving his general equations. This assumed relation cannot be adhered to exactly, but if the sizes of the girders at each floor are increased gradually from the top of the structure to the bottom, the final results will agree very closely with any of the results obtained by the so-called exact methods.

Case *A*-1 of the paper agrees very closely with the conditions assumed by Professor Yu. If the moment of inertia of the girders were 1.25 times that of the columns, the size of members, the story heights, and the span lengths will agree with these assumed conditions.

Applying the general equations introduced by Professor Yu to the fifth story of a 19-story structure of three equal bays with a unit horizontal load at each floor, the value of R_A for the fifth story is equal to $-$ 29.1579 lb and R_B is equal to $+$ 2.945 lb, which makes R_t for the fifth story equal to -10.10% and this result agrees very closely with the experimental data as given for Case *A*-1. In all stories the analytical sign values of R_A and R_B are of opposite sign, which agrees with the experimental data as given for Case *A*-1 of the paper.

[23] "Wind Stresses in Tall Buildings in Stresses in Statically Indeterminate Structures", by H. Yu, National Wuhan Univ., Wuchang, Hupeh, China, Second Edition, 1935, pp. 70 to 84.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

ADMINISTRATIVE CONTROL OF UNDERGROUND WATER: PHYSICAL AND LEGAL ASPECTS

Discussion

BY R. E. SAVAGE, ASSOC. M. AM. SOC. C. E.

R. E. SAVAGE[11], ASSOC. M. AM. SOC. C. E. (by letter).[11a]—A complete review of a new and important subject is presented in this paper. Mr. Conkling has handled the legal background of water legislation and Court decisions in an able manner. The paper points out some of the implications of public control of ground-water, assuming that such control will be exercised by the States in a wise and inclusive manner, in places where there is an economic justification and need for such a program.

The extreme variation in topographical, geological, and climatic conditions in the United States present a wide basis for the application of remedial legislation and governmental control of ground-water. In addition, there is the differing laws, customs, and application of laws by Civil Courts, resulting in the building up of a complicated structure. The author has emphasized all these phases and has shown the need for future control of underground water through the growing use of it for beneficial purposes.

The increase in population and the decreasing water supplies have brought the need of conservation of water to the fore. There is an ever-increasing demand for large supplies of water of relatively high standards of quality for industrial purposes in the centers of population in the United States. Large metropolitan areas have striven heroically to meet this demand by the construction of aqueducts bearing the supplies of mountain water-sheds a great distance. It is evident, however, that the economic and physical limit of these distant supplies is in sight. In the past few years engineers have witnessed a heightened discussion of the climatic cycles and their relation to the quantity of run-off water available for storage, irrigation, and domestic use, and the studies presented have been of absorbing interest. There is an increasing efficiency and excellence in the design of pumping machinery and motor-power

NOTE.—The paper by Harold Conkling, M. Am. Soc. C. E., was published in April, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1936, by Messrs. Joseph Jacobs, and W. D. Faucette and J. E. Willoughby.

[11] Asst. Engr., California Railroad Comm., San Francisco, Calif.

[11a] Received by the Secretary September 2, 1936.

equipment for the utilization of ground-water. There should be a corresponding advance in regulation and protection and in making the existing ground-water supplies available.

The writer has been greatly impressed by the need of regulation, by law, of the loss of wells in basins subject to harmful encroachment from salt water from adjacent coast lines and inlets, and also the need of eliminating the waste from virgin artesian supplies due to careless and destructive methods. There is no doubt that much benefit would result from the co-ordinated control of ground-water by public authority. After reading the paper, one is more than impressed with the need of distinct legislation for the various phases of water-supply control, wisely and skilfully drafted.

Any student of ground-water resources realizes the necessity of collecting information concerning the geologic nature, the history, and the yield of wells. In the private development of ground-water much of this information is lost, and these data can be collected and made available by future regulation.

In this comment on the physical aspects the writer has endeavored to keep in mind the difficulties besetting the progress of any comprehensive program of public control of ground-water, because of the diversity of the legal treatment of property relations in the ownership of water in the various States and political subdivisions. Mr. Conkling has ably clarified this phase of the situation. As stated under the heading, "Conclusion", it is incomprehensible that the control should be taken from the State, due to complex and extensive legal framework that has been constructed in the past as a means of controlling the surface waters.

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DISCUSSIONS

DYNAMIC DISTORTIONS IN STRUCTURES SUBJECTED TO SUDDEN EARTH SHOCK

Discussion

BY MESSRS. ARTHUR C. RUGE, AND H. M. ENGLE

ARTHUR C. RUGE,[9] ASSOC. M. AM. SOC. C. E. (by letter).[9a]—Engineers who have been complacently designing "quake-proof" structures by the statical method will find much to disturb their peace of mind in Professor Williams' excellent paper. The importance of considering the dynamics of structures is made clear, and the author has experimentally demonstrated the validity of theoretical analyses in a manner that should satisfy the most skeptical.

The author's attitude toward interpreting the results obtained from experiments in simple harmonic motion is well taken. To carry the interpretation beyond the first one or two cycles of the assumed simple ground motion is meaningless from the engineering standpoint and, as the results show, is unnecessary in the case of elevated water tanks. The writer was led to substantially the same conclusions that Professor Williams has stated by means of experiments of quite a different character, in which the water motion in the tank was not neglected.

There is only one statement in the paper (see "Conclusions") which seems to the writer to be open to question: "The results of the investigation further emphasize the desirability of designing a structure with a period different from that of an expected earthquake."

Unintentionally, this statement is misleading because there is no known single period or narrow range of periods to be expected in an earthquake, and because the practical problems of avoiding even the known earthquake periods are quite difficult in the case of tall tank structures.

If, for the sake of argument, one assumes that there exists in an earthquake a motion having the simple form expressed by Equation (1), in which the amplitude, G, is 1 in. and the period, $\frac{2\pi}{F_2}$, is 1.45 sec (exactly in resonance with the tank), then Professor Williams has shown that critical stresses will be reached during the first cycle of the ground motion. Now, suppose

NOTE.—The paper by Harry A. Williams, Assoc. M. Am. Soc. C. E., was published in May, 1936, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[9] Research Assoc. in Seismology, Dept. of Civ. and San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[9a] Received by the Secretary June 3, 1936.

it is decided to strengthen this structure in order to destroy the condition of resonance. Let the new stiffness be twice the original stiffness (which means the addition of 8 or 10 tons of steel to the framework); the new natural period of vibration will be $\frac{1.45}{\sqrt{2}}$, or 1.02 sec.

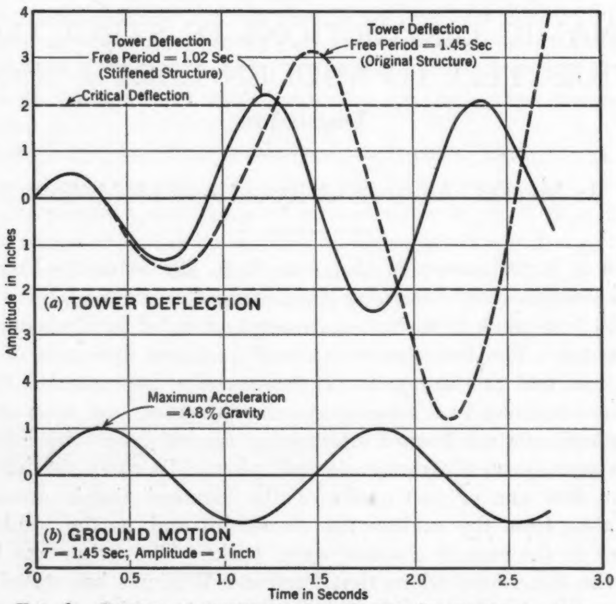

FIG. 8.—EFFECT OF DOUBLING THE TOWER STIFFNESS TO AVOID RESONANCE.

Fig. 8 shows the deflections of the stiffened structure as compared with the curve for the original structure in Fig. 7 (shown dotted in Fig. 8). It is seen that critical stresses are still reached during the first cycle of the ground motion. Hence, although the stiffening has limited the deflections, it has not prevented overstressing of the structure. Once the elastic limit has been exceeded, the subsequent behavior of the structure is largely a matter of chance and the calculated deflections become fictitious (that is, after $t = 1.25$ sec).

Now, suppose the earthquake motion contains, somewhere in it, one wave of 1-sec period with a 1-in. amplitude, as well as one wave of 1.5-sec period with a 1-in. amplitude (both of which are entirely possible). Doubling the tower stiffness would then have absolutely no beneficial effect and, even if the stiffness of the original structure were to be quadrupled, critical stresses would still be reached, and the stiffening would not have achieved its purpose. Furthermore, since the wave periods and amplitudes cannot possibly be predicted in advance for an actual earthquake, it clearly depends purely upon chance whether a moderate amount of stiffening will increase or decrease the safety of a given tank structure.

Measurements of actual tank-tower periods made[10] by the U. S. Coast and Geodetic Survey clearly illustrate how little is accomplished in avoiding known earthquake periods by moderate strengthening of structures in the 60 000 to 100 000-gal class. These excellent data should be of inestimable value in analyzing failures in future earthquakes.

The author rightly points out that consideration of the behavior of elevated water tanks beyond the critical distortion stage is of importance in attempting to explain many of the near-failures in the Long Beach, Calif., earthquake. In this phase of the problem chance again enters to a great extent. Even given some simple analytical method of attack, there is little justification for applying mathematical analysis very far beyond the yield point because of the strong probability of one side of the tower failing before the other side, which means almost certain collapse of the structure if the strong earthquake motions have not ceased by that time.

The Long Beach earthquake was indeed violent, but the most violent part was over in a few seconds. What might have happened to the many near-failures had there been a few more seconds of strong shaking can only be conjectured. There is reason to believe that the strong-motion phase of the San Francisco earthquake of 1906 was of longer duration than that of the Long Beach shock.

H. M. ENGLE,[11] ESQ. (by letter).[11a]—The Long Beach, Calif., earthquake of March 10, 1933, stimulated interest in the behavior of structures of various kinds. Elevated tank towers were adversely affected in many cases (although only three collapsed), and considerable attention has been focused on this particular type of structure. Due to the symmetry and structural simplicity of design, these towers lend themselves to model experiments and to theoretical attempts to explain what happened as well as to the determination of what the proper means of safeguard may be.

There has been a tendency on the part of those interested in shaking-table experiments and so-called "dynamic design" to consider the behavior of these structures as somewhat of a mystery, requiring complicated model study and theoretical experimentation. The writer has had intimate contact with the investigation, re-bracing, and repair of perhaps one hundred of such structures and also of a number of new towers. It is his belief: (1) That there is no particular mystery involved; (2) that a reasonable explanation of the damage can be made; and (3) that design to give assured safety against earthquakes is neither difficult nor economically impossible. Furthermore, no radical change in the proportions of towers, method of bracing, erection procedure, or past methods of design are necessary even if design in the past has been based on a static method of analysis. At present, there is no definite proof that the static method of analysis involves any more assumptions or uncertainties than the so-called dynamic method. A static analysis of damaged towers shows that they should have failed where failure occurred.

[10] *Bulletin*, Seismological Soc. of America, January, 1936.

[11] Engr., Board of Fire Underwriters of the Pacific, San Francisco, Calif.

[11a] Received by the Secretary August 17, 1936.

There should be no particular mystery about a structure being damaged under action of forces of a magnitude it was never designed to resist. The writer does not mean to infer that all tall towers of standardized design were wrecked or fatally damaged. Considering that they were braced for wind loads only, their behavior was surprisingly good, and not such as to warrant the idea that the entire conception of their design is faulty for earthquake resistance. They are the result of long years of development and, contrary to some expressed opinions, are skilfully detailed and proportioned not only to economize but also to utilize the metal involved to its fullest extent.

Following the Long Beach shock of March 10, 1933, the writer inspected in detail a large number of tank towers, some in the Long Beach area and many in other parts of California which had never been subjected to a severe shock. It was found that the towers had usually been erected with the bracing rods rather loose. Conversation with the erection foremen brought out the fact that in some cases this was done purposely as it was considered desirable that the tower be somewhat limber. How this idea originated the writer was unable to discover. In some cases the towers were considerably out of line. It is probable that correction of this defect alone would have reduced, somewhat, the damage to such structures in the Long Beach area. It is easy to see that, where loose, the rods would pick up their loads with a jerk. The tank would surge around on the tower, with possible heavy impact. Furthermore, twisting of the tank on the tower would be possible. Proper rigging with rods having good initial tension would reduce these secondary effects and would give a better chance for the structure to develop its strength effectively.

In the past most tanks have been built with hemi-spherical bottoms and a body about as high as, or somewhat higher than, the diameter. The tower legs are attached at the junction of the bottom and the cylindrical shell. The center of the mass of the tank and water is, therefore, a considerable distance above the column-to-tank connection. Under lateral movement the tank would tend to rotate in a vertical plane about the top of the tower.. With wave action developed in the tank appreciable stresses could be developed in the tower and bracing. With loose rods this tendency to rotate on top of the tower might add additional impact. Tanks with ellipsoidal bottoms have this defect in a less degree, and the writer believes their more general use in earthquake zones would be desirable.

In regard to tank design and behavior a number of problems may be discussed. Consider three tanks, each with a capacity of 100 000 gal. Let one tank be 75 ft in diameter and 3 ft high, with a flat bottom (see Fig. 9(*a*)). It could probably be shaken indefinitely without any appreciable lateral forces due to weight of water being developed. Friction beetween tank and water would be small. No appreciable wave action could develop. It would be much like a popcorn shaker. Consider another tank 12 ft in diameter and 132 ft high (see Fig. 9(*c*)). Under shock probably the entire mass of water, or most of it, would have to be moved. Consider as a third tank the common standard design approximately 22 ft in diameter and 24 ft high (see Fig. 9(*b*)). The action of this tank would probably be intermediate. A certain proportion of the water would probably move with the tank, and appreciable wave action

could develop. A strictly theoretical analysis of one of these towers to resist earthquake is probably impossible, being a complicated problem in wave action combined with various inertia effects, all produced by a chaotic earth motion impossible, as yet, to forecast. Perhaps Nature's shaking-table experiments on full-sized structures are the best guide. Actual disaster experience where properly interpreted is a fairly good foundation for basing future practice. One tower near Long Beach known to have been designed for a lateral force of 10% of the weight was undamaged in the shock of March 10, 1933. A static method of analysis was used with the lateral force applied at the center of gravity of the tank.

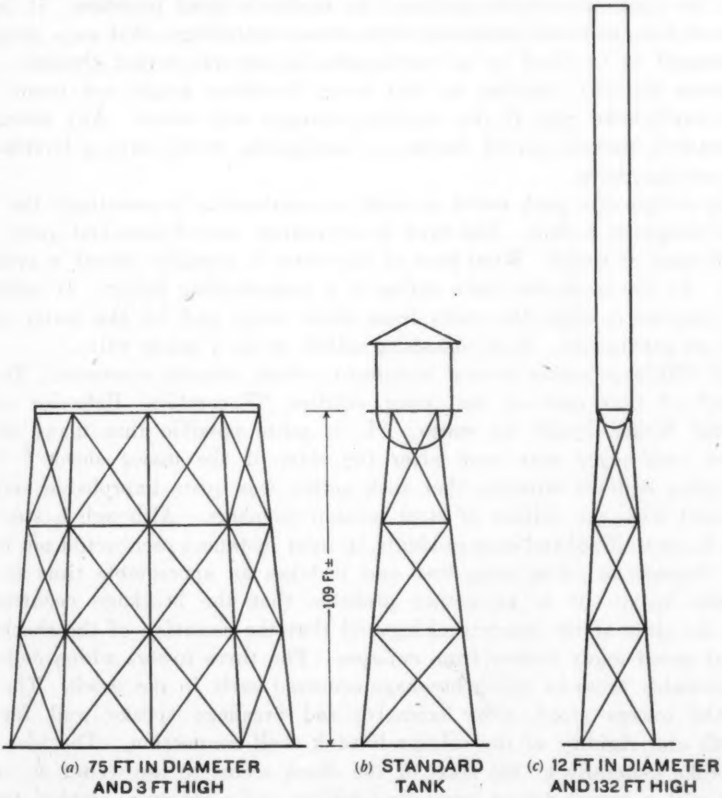

FIG. 9.

Proponents of dynamic design have laid considerable emphasis on the natural period of structures, generally assuming that there are certain dangerous and predominate natural ground periods which should be avoided. A period of from 1 sec to 1.5 sec has often been considered to be particularly dangerous and one to be avoided. As far as the writer can discover there is no certain basis for this belief. It is not concurred in by all outstanding seismologists. Periods from a fraction of a second to several minutes have

been recorded. For instance, both the Long Beach and Helena (Mont.) shocks had natural periods of only a fraction of a second. It would appear that the range of natural periods of structures is such that it is nearly impossible to dodge the possibility of a comparable natural ground period.

Resonance is often dwelt on in connection with behavior of structures. The danger from resonance has probably been exaggerated. The chaotic and changing nature of the earth motion alone takes away some of the danger. It is probable that the combination of tank and water has no set natural period of vibration. The movement of the water in the tank probably has a tendency to break up the natural period of the tank and water as measured under the slight movement produced by moderate wind pressure. It is also a proved fact, and one consistent with structural design, that as a structure is damaged or loosened in an earthquake its natural period changes. Few structures are put together so that some loosening might not occur in a severe earthquake even if the resulting damage was minor. Any change in a structure's natural period during an earthquake would have a tendency to break up resonance.

The design of a tank tower to resist an earthquake is essentially the same as the design of a dam. The tank is alternately moved into and away from a fluid mass of water. What part of the water is actually moved is problematical. In the tank also wave action is a complicating factor. It might be good practice to omit the roofs from these tanks and let the water splash out in an earthquake. Such splashing might act as a safety valve.

Mr. Williams makes several statements which deserve comment. Toward the end of that part of the paper entitled "Theoretical Behavior of an Elevated Water Tank" he states: "It is quite possible that many of the failures took place very soon after the start of the major shock * * *". The writer believes strongly that such action was quite improbable and not consistent with the failure of steel tension members. Although a steel rod might be crystallized and snap suddenly, in most instances such would not be the case. Stretching takes place first and it takes an appreciable time to jerk the rods apart. It is altogether probable that the breakage occurred at about the close of the heavy shaking and that the cessation of the shock was all that saved many towers from collapse. The three towers which collapsed were probably those in which breakage occurred early in the shock. The fact that the towers stood after extensive rod breakage speaks well for the strength and rigidity of the column-to-tank shell connection. The idea that rods broke generally at the start of the shock seems to the writer so inconsistent with a knowledge of usual steel failure and earthquake motion, that it is difficult for him to understand how it was ever proposed, except as an attempt to justify certain assumed dynamic action. If all the breakage occurred early in the shock why did such a large number of rods take "time out" to stretch? Under the heading "Conclusions", Mr. Williams states: "The results of the investigation further emphasize the desirability of designing a structure with a period different from that of an expected earthquake." Who is going to forecast what type the expected earthquake will be, or how much energy will be released, or exactly how the varying types of soil in a district

will respond? Who will guarantee that the period of the grand motion will be 0.2 sec, on 1.5 sec? Who will guarantee what time of year the shock may occur and whether the soil in a district is saturated and lubricated from long rains, or well drained after a long dry period? It seems to the writer that when some one can foretell what type the expected earthquake will be that assured earthquake prediction will be achieved. Some competent seismologists doubt whether such exact prediction will ever be possible. It certainly is not possible now.

Many of the measurements of natural periods of vibration of structures are of no practical value whatever unless closely correlated with the known features of structural design. There has been ample disaster experience to show that, as yet, no practical substitute has been evolved for ample and assured strength and unity of structure.

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DISCUSSIONS

FLOOD PROTECTION DATA
PROGRESS REPORT OF THE COMMITTEE

Discussion

BY MESSRS. JOHN C. HOYT, AND C. S. JARVIS

JOHN C. HOYT,[9] M. AM. SOC. C. E. (by letter).[9a]—Technical studies may be directed along two lines: (1) The collection of physical and other data; and (2) analyses and discussions based on these data. It has been noted that consulting engineers, planners, and promoters of various kinds appear to be more interested in the second line than in the first. It is also noted that the amount of work in connection with the second line usually decreases as the basic data increase. At present, much more attention is being given to technical studies than to the collection of basic data needed for those studies. That this is true is indicated by the report of the Committee and by statements of so-called experienced consultants.

Considering the many years that have passed and the many years that are to come, comparatively little information is available in regard to natural phenomena, and, as a result, it has been impossible to predict any future occurrence with certainty. This is especially true in regard to floods as illustrated by the following examples.

In 1881, the maximum flood discharge of the Sacramento River was estimated at 100 000 cu ft per sec. The report of the Commissioners of Public Works of California for 1905 contains what is generally known as the "Dabney report", prepared in 1904 by a board of engineers consisting of T. G. Dabney, State Levee Engineer, Mississippi; the late Henry B. Richardson, M. Am. Soc. C. E., member of the Mississippi River Commission; M. A. Nurse, Chief Engineer, State of California, Commissioner of Public Works; and the late H. M. Chittenden, Colonel, Corps of Engineers, U. S. Army, M. Am. Soc. C. E. This report outlines a project for the rectification of the Sacramento and San Joaquin Rivers and their principal tributaries. Paragraph 8 of the report states: "From the foregoing data, and with the allowance for the conditions described in the last paragraph, it is believed

NOTE.—The Progress Report of the Committee on Flood Protection Data was presented at the Annual Meeting, New York, N. Y., January 15, 1936, and published in February, 1936, *Proceedings*. Discussion on the report has appeared in *Proceedings*, as follows: April, 1936, by Messrs. Robert Follansbee, and LeRoy K. Sherman; and August, 1936, by C. R. Pettis, M. Am. Soc. C. E.

[9] Cons. Hydr. Engr., U. S. Geological Survey, Washington, D. C.

[9a] Received by the Secretary June 1, 1936.

that any scheme looking to the carrying of the run-off of the Sacramento watershed within the leveed banks of the river should make provision for a discharge of 250 000 second-feet below the outlet of Cache slough."

The report on the Sacramento flood-control project, revised plans, submitted to the Reclamation Board by W. F. McClure, State Engineer of California, in February, 1925, states (page 12): "The floods of 1907 and 1909 demonstrated the fact that floods more than twice as large as that for which the Dabney project has been designed should be used as a basis for a flood control project", and (page 18), "during cycles of wet years, general floods as much as 50 per cent greater than the 1907 flood may occur."

A complete history of the flood-control situation on the Sacramento River may be found in "Extract from House Report No. 616, Control of Floods on the Mississippi and Sacramento Rivers—Supplemental Report on Flood Control of the Sacramento River",[10] submitted by Mr. Charles F. Curry, of the Committee on Flood Control.

In designing the dam for the Sacandaga Reservoir which controls an area of 1 040 sq miles, 56 600 cu ft per sec was estimated as the 1 000-yr flood. In this estimate all existing data were used and interpreted in the light of the best judgment of the engineers in charge. During the floods of March, 1936, there were seven days when the flow approximated 55 000 cu ft per sec, and the actual peak was about 65 000 cu ft per sec.

The foregoing estimates were the best that could be made from the data available, and no criticism can be directed to the engineers who made them or to the organizations represented. They are presented only as an illustration of the futility of attempting to use any substitute for actual stream-flow data.

Much money and effort are being expended in working over and over old data, and as a result many generalized reports have been issued which, in the main, "do not arrive." The fact that information on past natural phenomena is so meager is deplorable. In the interest of those who follow, more attention should be given to insuring the collection of the fullest information in regard to passing natural phenomena which will be essential for future wise planning.

C. S. Jarvis,[11] M. Am. Soc. C. E. (by letter).[11a]—The broad understanding and comprehensive, clear-sighted views concerning flood data and related problems, as shown in this progress report, are consistent with the ideas expressed by individual members and developed during successive conferences and consultations with reference to subject-matter and procedure in this field. The Committee had been appointed and delegated by the Society to assist in the guidance of investigations reported[12] by the U. S. Geological Survey. Naturally, this work included such preliminary studies as reviews of the best-known methods of summarizing, plotting, and analyzing flood records; the

[10] To accompany H. R. Doc. No. 14 777, 64th Cong., 1st Session, 1916.

[11] "Hydr. Engr., Soil Conservation Service, Washington, D. C.

[11a] Received by the Secretary August 31, 1936.

[12] "Floods in the United States—Magnitude and Frequency", *Water Supply Paper 771*, U. S. Geological Survey.

determination of what classes of data are the most significant and representative; and the adoption of suitable forms for various tabulations.

Throughout the period of association on this work, the members of the entire staff engaged on the project were aware of the able leadership and earnest team-work maintained by the U. S. Geological Survey and the Committee in both administrative and technical matters, and in both advisory and supervisory capacities. This experience has confirmed the writer's opinion that the analysis and interpretation of available hydrologic data should be a continuing responsibility of the governmental agencies charged with the collection of such information, and, particularly, this should be a prescribed duty of those organizations best equipped for its authoritative rendering in usable form.

Determined efforts to bring the most notable flood records to date for *Water Supply Paper 771* were ruthlessly offset by the catastrophic floods of the following years, 1935 and 1936. These floods have established new records on a number of river systems and have provided further proof of the disparity between brief or fragmentary hydrologic records and the actual potentialities of such streams for flood flow.

In his eagerness to accumulate hydrologic records, the civil engineer is prone to overlook the fact that the flood potentialities, probabilities, and frequencies are the more valuable and significant data, one step removed from the basic records, and attainable only through competent analysis and interpretation. Notable progress in this direction was achieved by the Boston Society of Civil Engineers after investigating the New England floods of November, 1927. The resulting total run-off formula, $Q = C R \sqrt{A}$, expressing peak flow in cubic feet per second as a function of rainfall and drainage area as modified by the coefficient, C, was associated with the idea of a fairly constant base for hydrographs of storm run-off at a given station, the peak varying with total run-off. The later development, extension, and application of this concept by L. K. Sherman and Merrill M. Bernard, Members, Am. Soc. C. E., resulting in the unit hydrograph method and its corollary relationships, have further simplified the problem.

Following the most promising trends for determining flood probabilities and potentialities, the hydrologist is concerned primarily with: (*a*) Rainfall intensities and cumulative depths for various units of time; (*b*) the portions of such precipitation likely to appear as run-off; and (*c*) the peak discharge and the total flood volume for which provision should be made, either in channels, detention basins, retarding features, storage reservoirs, or combinations of them. Successive steps along lines of accepted procedure will readily define, thereafter, the flood magnitudes likely to be equaled or exceeded in given periods, as 20, 50, or 100 yr. Some leaders in this field of investigation prefer to deal with the average magnitudes of expected floods, thus adopting somewhat higher discharges, corresponding to about twice the periodicities of the floods "equalled or exceeded".

With these derivations completed from adequate basic data, it is often discovered that the probable flood potentialities of adjacent or intervening drainage basins, with but meager if any hydrologic data, have emerged from

the unknown realms to the known, or, at least, to the "twilight zone". By this process the writer had been led to expect more severe flood flows than were of record in Central New York State, and had long been at a loss to explain the relatively low flood peaks there prevalent as compared with the adjacent Ohio Valley and the New England States. Subsequent to that investigation, the 1935 and 1936 floods in that region have measured fully up to expectation, or somewhat beyond in some instances, especially where the ratings on Myers scale exceeded 50 per cent[13].

By all means, the recommendations of the Committee should be indorsed and followed in detail, utilizing existing facilities and supplementing them as found necessary, and the same members should be retained for active duty in advisory capacities in order to insure realization of their express d aims and concepts.

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[13] "New York State Flood of July, 1935", *Water Supply Paper 773-E*, U. S. Geological Survey, pp. 256–263.

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DISCUSSIONS

PRINCIPLES TO CONTROL GOVERNMENTAL EXPENDITURES FOR PUBLIC WORKS

FIRST PROGRESS REPORT OF COMMITTEE OF ENGINEERING-ECONOMICS AND FINANCE DIVISION

Discussion

BY MESSRS. J. K. FINCH, ALBERT GIVAN, EDWARD HYATT, CLARENCE H. KROMER, A. G. MOTT, AND C. S. POPE, AND JOSEPH JACOBS.

J. K. FINCH,[22] M. AM. SOC. C. E. (by letter).[22a]—Few engineers, who have given thoughtful consideration to this subject, will question the necessity of establishing a reasonable program for the investment of public funds such as the Committee suggests.

The members of the Committee apparently intended that the principles enunciated in their progress report should apply to public works policies and practice in general, although they have set up these principles "to guide a Federal Public Works Program." To-day, of course, Federal activities and funds are the main element in the public works field and constitute the chief present activity of the construction industry. Under normal conditions, however, Federal works account for only 10% of public construction. State expenditures average about 15% (mostly highways), counties and towns take, perhaps, 25%, but it is the many municipalities, scattered all over the country, which normally spend at least one-half the money used annually for public works. Under normal conditions, therefore, the principal public works problem is concerned with municipal rather than Federal activities.

NOTE.—The Progress Report of the Committee of the Engineering-Economics and Finance Division on Principles to Control Governmental Expenditures for Public Works, was presented at the Annual Meeting, New York, N. Y., January 15, 1936, and published in February, 1936, *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: April, 1936, by Messrs. Edward W. Bush, Fred Lavis, and Horace H. Sears; and August, 1936, by Messrs. Ivan C. Crawford, and Samuel B. Folk.

[22] Renwick Prof. of Civ. Eng., and Director of Summer School of Surveying, Columbia Univ., New York, N. Y.

[22a] Received by the Secretary April 30, 1936.

Thus, while the Committee's proposal for a Federal Board of Engineering Review could be developed in a similar form for State works, it is difficult to visualize how it could be further extended to cover municipal undertakings. Thousands of such boards would be required, and many municipalities could not afford to secure the highest type of engineering advice which is an essential to the proposed plan.

In fact, such a board, at best, must be purely advisory—approval and appropriation of public funds rests with Congress, with the State legislatures, with municipal councils, or they may be dependent on general referendums. Such a board cannot have any final authority. Its purpose must be primarily to insure that proposed projects are given adequate study and economic analysis by a competent group of unbiased experts. It is public opinion which in the end shapes the policies of representative government.

In the past, of course, Congress has listened attentively to the advice of such a board, but it is obviously necessary for a board of this type to develop non-partisan methods of bringing the results of its studies to the attention of public officials and to the electorate. In the end reliance must be placed on informed public opinion. The Government Printing Office is already a "morgue" for thousands of public documents and, without final authority, and lacking any effective means of bringing its findings to public attention, such a board, even if limited to Federal and State works, would simply add new volumes to the existing collection of forgotten records. A proposal very similar to that of this Committee has recently been made by Mr. C. Hartley Grattan[23] who points out the difficulties of making such a board an effective instrument in directing public policies toward sound economic growth and the sensible investment of public funds.

Another point in the Committee's progress report demands comment. In preparing this report, the Committee evidently had two major problems in mind, one dealing with policy and one with practice. It has emphasized the necessity of "sound economic considerations" and has also set up standards for "evaluating the benefits of public undertakings."

Apparently, the Committee believes that the present policy of undertaking public works in competition with private enterprise on an unfair, uneconomic, and destructive basis should be stopped. When Federal funds are used to build hydro-electric plants and power is sold to the public at rates far below cost through the simple expedient of marking off to non-existent navigation, or some other equally visionary use, a large part of the capital expenditure, the Government is no longer co-operating in the economic rehabilitation of the nation. It is not only undermining legitimate business enterprise, but it is also destroying important sources of public income. When funds are "granted" to municipalities for the construction of municipally owned power plants to replace existing private plants that are furnishing good service at reasonable cost, as, for example, at Iowa City, Iowa, or as was proposed in New York, N. Y., the local power users are being subsidized at

[23] "Working for the Government", *Scribner's Magazine*, February, 1936, p. 100.

Federal expense in a movement which cannot fail in the end to demoralize the economic life of their community.

No thoughful man will take issue with the ideas of the Committee on this subject, but it does seem extremely doubtful whether even a Supreme Court can alter a policy which depends for support on local public opinion. "The money is there", says the local power user, looking no further than the monthly electric bill, "and we might as well take advantage of the possibility of lower rates." The nation has had sufficient experience with attempts to secure temperance through legislation, so that corresponding efforts to legislate economic common sense will be regarded with suspicion. As Daniel Webster pointed out more than 100 yr ago, in a famous address on internal improvements, this tendency to wasteful and destructive spending is one of the great dangers of so-called representative government. It is inherent in that form of government and cannot be cured by legislation.

But the Fed. Gov't need not subsidize

The efforts of any type of Engineering Board of Review, therefore, must be directed toward informing the people's representatives on the "pros" and "cons" of proposed expenditures and toward securing public support for sane policies of public investment. The problem is one of information and education and the success of such a board will rest in large measure on the effectiveness of the organization which is developed for broadcasting the ideas and ideals of the Board.

Finally, on the matter of "practice" all engineers will agree that an evaluation of benefits, as full and complete as the best of modern methods permit, forms an essential first step in deciding whether a proposed public work is economically justified or must be looked upon as a luxury to be afforded only if the state of the public treasury is such that a financially unproductive expenditure can be undertaken. The constructive suggestions of the Committee along these lines form a most valuable part of the report. Here again, however, much must be done in educating public opinion to the fact that financially unproductive expenditures mean increased taxes. A large number of people to-day seem to feel that the Government finds funds in some mysterious way, and the question of whether these funds are spent on revenue-producing works or for public luxuries is unimportant. Every borrowed dollar spent will cost the taxpayers at least $2 or $3 in interest and amortization charges before the debt is cancelled, but past records seem to indicate that the public will take no action until the burden of ever-mounting taxes finally results in protests against unnecessary and unwise Government spending. Even the Committee's simple and obviously sensible principle of evaluating benefits before spending public funds, therefore, will fail of acceptance until a large amount of pioneer work has been done in educating public opinion.

It is to be hoped that the Committee will develop more fully the plan for Engineering Boards of Review, and will publish its final report in a form which may be widely distributed to members of the Society and to others interested in this vital problem.

ALBERT GIVAN[24], EDWARD HYATT[25], CLARENCE H. KROMER[26], A. G. MOTT[27], AND C. S. POPE[28], MEMBERS, AM. SOC. C. E. (by letter).[28a]—Objections to this report should be registered on the grounds that: (1) The subject raised is not a proper one for a Division of the Society upon which to issue a pronouncement; and (2) even if it were in a field properly subject to civil engineering review, the Division should not undertake to advocate principles that restrict public works to those which meet certain prescribed economic standards.

The reason for Objection (1) is that, far from being non-partisan, the report deals with one of the most outstanding partisan questions now before the people of the United States, and that it is entirely outside the functions of any official sub-division of the Society to participate in partisan activities.

Referring to Objection (2) it is quite beyond the proper functions of an engineering society to advocate politico-economic criteria. Like engineering principles, economic principles are, and must be, elastic. They should be applied to meet the varying conditions that arise, whether such conditions are physical or social in character. To attempt to establish a fixed principle for their observance is to admit either weakness or an ignorance of their fundamentals.

When war prevails, or serious emergencies arise, economic principles and engineering practices are modified and made applicable to the conditions to be met. A fixed standard of principles or practice to meet all exigencies cannot be promulgated.

In reviewing the seven principles enunciated, the writers find this report to be partisan and contradictory. For example, if the non-partisan nature of the Federal Agency proposed in Principle (1) is to be made secure, some of the other principles will need considerable revision. This is not to say that there is no potential value in some of these principles, but they are distinctly partisan in their present form and a thorough revision will be necessary to make them non-partisan.

There is no more partisan principle before the people to-day than the controversy between advocates of private and public ownership, as witnessed by the fact that it is a major factor in the present national political campaign. The report simply lends fuel to that controversy. Certainly, the Division should not wish to plunge into politics in defense of either side as the result would injure the reputation of the Society as a whole, and the reputation of all its members.

The civil engineers of the United States enjoy a high reputation and they can, and must, participate in an important manner in any Federal construction

NOTE.—At the meeting of the Sacramento Section of the Society this discussion, in formal style, was presented by the writers, constituting the Public Relations Committee of the Section (Mr. Kromer, *Chairman*). After presentation, the report of the Section Committee was approved and adopted by the Sacramento Section, and the Secretary of the Section was authorized to transmit it to the Society for publication.

[24] Gen. Mgr. and Chf. Engr., Sacramento Municipal Utility Dist., Sacramento, Calif.

[25] State Engr., Sacramento, Calif.

[26] Prin. Structural Engr., State Dept. of Public Works, Div. of Architecture, Sacramento, Calif.

[27] Director, Valuation Div., State Board of Equalization, Sacramento, Calif.

[28] Chief Constr. Engr., Calif. Div. of Highways, Sacramento, Calif.

[28a] Received by the Secretary April 23, 1936.

program; but they will only do themselves harm by participating on either side in a bitter political battle.

Principle (2) proposes that "only those projects should be included that will answer a definite and well established public need which cannot be met reasonably by private capital."

Principle (3) proposes that "no project should be included which would duplicate, or impair the value of, existing property and facilities that afford adequate service to the public at reasonable rates." The enunciation of Principles (2) and (3) supports the contention of the writers that the report is of a partisan character. From a practical standpoint all the principles enunciated are entirely concerned with policy. Possibly such policy does not lead to the question of appropriations but certainly it concerns the policy that should govern expenditures made under appropriations, and, therefore, the policy which would guide the wording of appropriation bills themselves.

Principle (3) is particularly objectionable. In the discussion supporting it there appears the statement that new work projects should not be undertaken unless existing facilities are being used to capacity. If applied strictly, this principle would be indefensible in the United States, which has always maintained the philosophy of free play of competition. For instance, under this principle, Federal aid would not have been given to the railroads during their construction periods since service was being maintained by the then existing primitive methods of transportation. Carrying the illustration further, under Principle (3), somewhat extended, the Reconstruction Finance Corporation would not have made railway loans in recent years since competitive roads certainly were not being used to capacity. Reasonable restraint on duplication of facilities is in order, but the principles enunciated in the report would be destructive of initiative and would penalize the public by perpetuating obsolete facilities.

The principles enunciated in the report, if followed to a conclusion, would have restricted progress in many meritorious projects of a public character, in the past. Principle (6) would have eliminated most relief projects recently prosecuted and no body of citizens concerned with the unemployment problem would unequivocally concede this principle in periods when unemployment is serious.

Principle (7) with its explanatory discussion, lists in detail the items entering into the cost of the project both direct and indirect. However, the much more difficult but equally important phase of resulting benefits is dismissed with a single brief and inadequate sentence. Here is a phase which deserves much study and on which the Committee of the Division might render useful service. The distribution of benefits flowing from a Federally financed project is a subject on which light is needed.

The present is a time of social unrest. Scientific and mechanical invention and progress through the last generation, have outstripped social progress leaving problems to be solved, particularly unemployment created partly by technical advancement. Unemployment is a real problem and not a myth. Very likely the present is a time of social change and re-adjustment. Great wisdom will be required to overcome these problems.

It is urged that the Division does have a duty to perform; it should, therefore, formulate a comprehensive statement outlining the scope of engineering studies that should be made, and the type of report that should be submitted to the public before action is taken to authorize any public works project (other than emergency projects). The Division would render a very real public service if it would advocate and insist that thorough and comprehensive studies be made available to the electorate and appropriative legislative bodies, on all appurtenant engineering and economic facts relating to a proposed project, so that the decision to approve or disapprove such projects could be made intelligently in the light of all such facts. The Division should sponsor the development of a set of standards outlining the nature and scope of such studies and should provide particularly that all benefits and burdens involved therein be sufficiently set forth. It should not attempt to determine social policies, or to establish rules for governmental administration; to attempt to do so would be presumptuous and would only tend to make the Society a partisan agency to the permanent injury of its present prestige.

In conclusion, it may be stated that although there is potential constructive value in this report, it is partisan and contradictory in several places. Under the guise of a non-partisan statement, it is intensely partisan on one of the foremost political issues of the day. If this Committee report is endorsed by the Division it might appear to those unfamiliar with the organization, to convict the Society as a whole of hypocrisy as well as political bias. In its present form, the report will have a harmful effect upon the Society and the Engineering Profession, and it is hoped that the final report on this subject will be a truly technical and non-partisan one. If possible, it should have a more comprehensive scope if it is to be of aid in solving the great problem which the country is facing.

JOSEPH JACOBS,[29] M. AM. SOC. C. E. (by letter).[29a]—From correspondence following submittal, to members of the Division, of the Committee's tentative report of November, 1935, the writer anticipated some revision of that report before publication. As presented, however, it is, except for minor editorial changes, identical with the original tentative report, both as to statement of principles and as to the "Remarks" or arguments supporting those principles. Although the writer, in the main, regards the enunciated principles as sound, if wisely interpreted and applied, he finds himself not in accord with certain phases of the supporting arguments. If the principles are enacted into law as the basis for future Federal action in the matters with which they are concerned, and if the Committee's specific and implied applications of those principles are attempted, then the final outcome is highly uncertain because, it seems to the writer, in the Committee's presentation of its case there is an unfortunate confusion of important issues. The following discussion relates mainly to the Committee's "Remarks" in elucidation of the principles it sets forth, rather than to the principles themselves, and it is drawn largely from the writer's comments on the Committee's original tentative report.

[29] Cons. Engr., Seattle, Wash.

[29a] Received by the Secretary May 20, 1936.

There appears to be a confusion, running continuously throughout the report, as to certain fundamental considerations that should have been clarified by a preamble indicating exactly what the Committee was attempting to do. Was it attempting to outline a Federal policy adapted to the depression conditions that have obtained during the past several years or one adapted to normal conditions. As the country is now emerging from the depression, and as the depression must be regarded as an abnormal condition, it would seem unquestionable that any permanent, Federal, public works policy should relate to, and be predicated upon, normal economic conditions—with, perhaps, some special provisions for adaptation to emergency conditions when necessary. Some of the confusions noted in the Committee's presentation are:

(*a*) A failure to distinguish between normal conditions and emergency conditions. Apparently, the report in the main, is framed for emergency rather than for normal conditions. The writer feels, strongly, that it should have been framed, primarily, for normal conditions.

(*b*) A failure to distinguish between Federal and non-Federal public works. Much of the report deals with the latter although, by title, it is intended to refer only to the former. Normally, the Federal Government has nothing to do with non-Federal public works although, in respect of over-all taxation, future planning should take cognizance of both forms of expenditures and, to that extent, there should be some co-ordination of the two.

The acute emergency situation that has heretofore prevailed has now passed, and the country should now plan for a future of normalcy because it is already in the midst of recovery. Federal grants, subsidies, and loans for non-Federal public works are all abnormalities that were brought into being by the exigency of the depression and they should be abandoned at the earliest date practicable—just as soon as normalcy, or approximate normalcy, has been restored. The lesser governmental units (States, counties, cities, etc.) should return, as soon as possible, to the ordinary procedures for financing their public works projects and should not continue to rely on the beneficence of the Federal Government. An indefinite continuance of such paternalism will tend, inevitably, to weaken the morale and the sense of financial responsibility of those lesser agencies. It might also easily lead to a definite Federal encroachment on State rights and that is a consummation not to be desired.

It is not intended, by the foregoing comment, to imply that there should be no co-operation between Federal and non-Federal agencies concerned with public works. There is definite need for co-operation between these agencies, particularly as regards public works in which there is a multiple interest, but the prerogatives and responsibilities of these separate agencies should be clear, distinct, and properly safeguarded. The bearing of public works expenditures upon possible limitations of public credit and of tolerable taxation make Federal and non-Federal public works more or less inter-related. In this connection it is of interest to note that in the country as a whole, for the period, 1920 to 1932, expenditures for Federal public works averaged only one-eighth that of non-Federal public works and that the two combined

were only 37% of total construction expenditures; or, in other words, that the total of public construction was but little in excess of one-half that of private construction.

It is now desired to comment, more in detail, on the "Remarks" sections of the Committee report as to each of the specific principles which it lists as basic to a Federal public works program. Before proceeding further, however, the writer would suggest, for the future planning of public works, an additional basic principle which he deems important, namely, that of budgeting. Planning and budgeting must go hand in hand. Without the latter the former may easily over-reach itself with a resultant straining of public finance and an endangering of public credit. Conservation of financial resources is just as essential in national economy as conservation of natural resources; and, by conservation is meant a wise and timely development and utilization of resources—that is, a conservation policy of neither bottling up nor of wantonly wasting such resources. Public works policies and programs are definitely related to these considerations.

Principle (1).—Reference to the Supreme Court in Principle (1) is unfortunate. No public works agency that may be created can possibly be endowed with a character or an authority comparable to that of the Supreme Court—nor should it be. The existing Federal departments would strenuously oppose, as would also the Congress, any serious effort to create a new organization with absolute final authority over all Federal public works activities. A technical, advisory body with a broad view and understanding of public needs, and a broad knowledge of public finance and budgeting, could indeed be helpful, but it need not and should not be a Court of last resort respecting public works, and thus usurp a prerogative of the Congress. A well-informed, technical advisory body would be more serviceable, and more effective, in securing approval of long-range public works programs, than an "Authority" such as is apparently contemplated by the Committee.

The name, "United States Program Authority", as suggested in the report, is not definitive. There may be all kinds of Federal programs for all kinds of activities, and there is no indication in this proposed title that it is intended to deal only with public works. As stated in the preceding paragraph, it should not in any event be an "Authority", but rather an advisory and co-ordinating body that would handle no construction work itself, but would leave that activity to existing Federal construction agencies. In determining the form of this proposed new agency, and its functions, serious consideration should be given to the following:

(*a*) A National Planning Board comparable to, and working in close co-operation with, Regional and State Planning Boards. Something of that nature is now in existence in the National Resources Committee (originally designated, National Planning Board), but its permanent status and its financial support are not yet adequately assured, although measures toward that end are now before the Congress.

(*b*) A permanent Public Works Administration (PWA), as at one time suggested by the National Resources Committee, with, no doubt, powers and duties somewhat different from those that now obtain for that organization.

(*c*) The Federal Employment Stabilization Board which has been doing a splendid work and which might be made effective for all those functions with which the Committee's proposed new agency would be concerned.

The first paragraph of the Committee's "Remarks" clearly implies a continuance of the policy of Federal aid for non-Federal projects. As already stated (see Fifth Paragraph), that Federal policy was adopted solely to meet an acute economic emergency, and it should be abandoned with the passing of the emergency. It should not become a permanent Federal policy.

Principle (2).—It is suggested that all reference to communist, fascist, or capitalist types of governmental organization should have been omitted from the "Remarks" pertaining to Principle (2). These references are wholly non-essential to the Committee's clean-cut initial statement as to what is meant by "public need." Such extraneous matter detracts from the directness of the initial statement and raises issues not pertinent to the question in hand as, indeed, the "Remarks" themselves specifically assert. The statement that projects should be given priority in accordance with their relative importance might also be omitted because it is fully covered in Principle (4) where its discussion properly belongs.

In the Committee's Sub-Paragraphs (*A*) and (*B*) there is persistent confusion of Federal and non-Federal public works, and of emergency and normal public works activities, to which reference has already been made. Normally the Federal Government has nothing to do with city halls or with parks, schools, hospitals, etc., of the lesser governmental units, and this report, per title, is intended to deal only with Federal public works. It has already been indicated that, in the writer's judgment, there should be co-operation between Federal and non-Federal agencies in respect to public works in which there is a multiple interest (flood control, river and harbor improvements, highways, etc.); and also some mutually agreeable and practicable co-ordination in respect to total public expenditures; but beyond these the Federal Government should not concern itself with, nor presume to inject itself into, local affairs.

Principle (3).—The writer agrees heartily with Principle (3), wisely applied, but he does not agree at all with the statement that it "would exclude projects of reclaiming arid land or draining wet lands unless all other cultivable lands in the country were being used to capacity." In the first place, reclaimed arid land, properly irrigated, may, and in fact generally does, provide a more valuable and more profitable agriculture than ordinary cultivated land. To contend that there should be no development of the former until all the latter has been exhausted is not only unsound economically, but it condemns unjustly much of the Western domain, largely dependent on irrigation, to non-development for an indefinite period of years. Secondly, for a wholesome national growth, and also for defense purposes in a military

sense, there should be encouragement of a more or less uniform development throughout the United States. Moreover, there should be provision, within reason, for people making their homes and livelihood in places of their own choosing, and they should not be regimented or compelled to live in localities against their wishes and desires. It is just as logical to state that no more houses shall be built in Denver, Colo., as long as there are vacancies in Philadelphia, Pa., or in Boston, Mass., as to contend that no irrigable lands should be reclaimed as long as there are non-irrigable lands not yet fully developed. Finally, with many sub-normal and drought-stricken areas going out of cultivation, with continued growth of population, and with the extant, gross exaggeration as to excesses of agricultural production, there is need for new agricultural development, particularly in the West.

The writer cordially agrees with the proposition that it should not be the policy of the Federal Government to build cement plants to meet the requirements of its extensive construction operations, but the implications of the statement by the Committee are too broad. The Government should not build such plants except as a last resort but it should not be estopped therefrom if fair bids for cement cannot be obtained from private establishments. The Committee should look into the history of cement prices tendered for the Roosevelt Dam, in Arizona (which, upon both the first and second calls for bids, were exorbitant), and note how the Government felt compelled, in self-defense, to erect its own plant for that construction, and did so with a resultant material saving in cost. The statement of the Committee should be qualified.

That the replacement value of an old plant should always be added to the cost of the new one, even if the added cost is not charged to the consumer, is at least a debatable question. Condemning or burdening a procedure that might result in the destruction of an old and uneconomical plant, and the construction of a new and economical plant, is also debatable economic philosophy. Just what does the Committee mean by "a fair proposal to write off some of the remaining investment in the old plant"? Industrial advance does not lie in the direction of perpetuating the uneconomical, and the principle of competition should be safeguarded in respect to those industries that are properly competitive within themselves. Progress requires that the old and less economical shall yield to the new and more economical even if a loss may thereby be imposed upon the former If the Committee has in mind some form of subsidy for the old and uneconomical, when supercession by something new and more economical is threatened, then it is proposing a policy subject to the possibility of serious abuse against the public interest unless it is very carefully safeguarded.

Principle (4).—Why should the Committee, in its "Remarks" pertaining to Principle (4), assume, or encourage the idea, that the United States Government, as a permanent Federal policy, should underwrite all, or any part, of purely local, non-Federal public works projects. As the writer has already emphasized, that has been wholly an emergency provision during the period of depression and one that should be abandoned as soon as normalcy has been restored. Local governments should return as soon as possible to normal

procedures for financing their public works enterprises and should not continue to rely on the Federal Government. Continuance of such paternalistic treatment will inevitably tend to weaken the governmental morale of States and municipalities. These local agencies should be compelled to "stand on their own feet" and should not be supported by the Federal Government except in times of real emergency.

Principle (5).—In the Committee's "Remarks" accompanying Principle (5) there is again a lack of clear distinction between emergency conditions and normal conditions. For emergency conditions, proximity to centers of unemployment would weigh heavily in selecting projects for construction. For normal conditions, however, other criteria would predominate—mainly, their definite, long-time economic need and the prospect of their financial success—and it is to normal conditions that a Federal public works program should now apply primarily.

Principle (6).—On the basis of meeting the emergency conditions to which the "Remarks" on Principle (6) refer, the supporting argument is entirely correct. The principle itself, however, has even stronger application to normal economic conditions, and it is mainly in respect to the latter that it should now be formulated and applied.

Principle (7).—The writer regards Principle (7) as eminently correct, particularly in its application to public utilities, and he also concurs in the Committee's argument supporting it.

The concluding comment which the writer would make respecting this report is that prior to approval of any set of principles intended to govern the conception and prosecution of a Federal public works program, and prior to any effort being made to secure the enactment of legislation that would effectuate such a set of principles, the Committee should modify its report in several particulars and re-submit it for consideration. It is believed that the principle of budgeting should be added to those already listed in the Committee report; that Principle (1) should be recast with the omission of any reference to the United States Supreme Court and with the addition of a clear definition of the duties and functions of the proposed new agency; and, finally, that the supporting arguments elucidating the principles should be revised materially to the end that confusion of purpose and objectives be eliminated, the revisions, perhaps, to be in the general direction pointed in the foregoing comments. If and when a body of principles, as herein contemplated, is finally submitted by the Division to the Board of Direction for approval and is then approved by the Board, much additional work will be required, of course, to make such principles effective.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

WIND BRACING IN STEEL BUILDINGS

FIFTH PROGRESS REPORT OF SUB-COMMITTEE NO. 31 COMMITTEE ON STEEL OF THE STRUCTURAL DIVISION

Discussion

BY MESSRS. ROBINS FLEMING, L. E. GRINTER, CHR. NOKKENTVED, AND I. WOUTERS

ROBINS FLEMING,[32] ESQ. (by letter).[32a]—The Fifth Progress Report of the Committee is of exceptional value throughout.

Prominence is given to wind force on rounded and sloping roofs. A long array of experimenters is cited. The suction on the leeward slope is emphasized. Although American textbooks have not ignored this phase of wind action, they have not given a method of computing suction effects. Designers, following the textbooks, have been accustomed to make provision for direct pressure on the windward slope—of course, at some time or other, either slope may be the windward slope. There is no doubt now but that suction exists on the leeward slope; for roofs nearly flat it exists on both slopes. In the past the writer has advocated the Duchemin formula for determining pressure on sloping roofs. In view of the accumulated evidence of the inadequacy of this time-honored formula he is obliged to change his former views. Until something better presents itself, he would recommend that the pressure and loadings suggested in the report be followed.

The Committee does well in calling attention to the paper[29], "Tall Building Frames Studied by Means of Mechanical Models", by Francis P. Witmer, M. Am. Soc. C. E., and Harry H. Bonner, Esq. The purpose of the study was to learn trends rather than true quantitative values of reactions for varying proportions of parts. The conclusions are important. The paper by Messrs. Witmer and Bonner should be read and studied in connection with the report.

NOTE.—The Fifth Progress Report of Sub-Committee No. 31, Committee on Steel of the Structural Division on Wind-Bracing in Steel Buildings, was presented at the meeting of the Structural Division, New York, N. Y., January 16, 1936, and published in March, 1936, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the report.

[32] With American Bridge Co., New York, N. Y.

[32a] Received by the Secretary April 3, 1936.

[29] *Proceedings*, Am. Soc. C. E., January, 1936, p. 3.

Comment will be made on the statement regarding the use of the cantilever and the portal methods for moderate ratios of height to breadth. The report states "although sufficient aggregate strength may be secured in this way there is no assurance that a balanced design will be attained, or that unduly high local stresses may not exist." Notwithstanding adverse criticism the average engineer, for the class of tier buildings he is called upon to design and with the time at his disposal, will continue to use one of these two conventional methods. What are their limitations? Of the 5 000 buildings in the United States, ten stories or more in height, about 400 are of twenty stories or more. It is only occasionally that a structure is on a narrow plot. Among such are the 42-story North Michigan Avenue Building, in Chicago, Ill., on a plot 100 by 125 ft; the 43-story building at 20 East 40th Street, New York City, 503 ft high on a base, 71 ft 3 in. by 147 ft 4 in.; and the Medical Tower Office Building, in Philadelphia, Pa., rising to a height of 363 ft, from a base of 44 ft 8 in. by 75 ft 7 in. The tower portions of other buildings present similar ratios of base to height.

Incidentally, it may be noted that the 20-story bent analyzed by Messrs. Wilson and Maney in their well-known monograph on slope deflection has a width of only 62 ft, which is a ratio of base to height of 1 to 4.2. The writer would not hesitate to use either conventional method when the ratio of base to height did not exceed 1 to 3.5, or, perhaps, 1 to 4. At the same time, and after members are proportioned, he would examine the design by one of the methods considered more exact. Just what method he is not prepared to state. The present report commends the "Goldberg method", and also mentions favorably the "Grinter method." A few years ago, David Cushman Coyle, M. Am. Soc. C. E., wrote[33]: "When the importance of the project justifies the use of the best engineering talent, it seems evident that Method No. 7 [the Spurr method] is the only one that can be said to meet the requirements, both because it provides for keeping the floor plane and because it does not allow the engineer to forget the deflections." The irregular spacing of columns, so common in apartment houses, the break in continuity in columns due to assembly rooms in office buildings or ball-rooms in hotels, the setback requirements of municipal building codes, all present need for careful study, whatever method of wind stress determination is followed.

In his comments on the Second Progress Report[34] the writer called attention to the habit of college professors of under-rating the time required to solve a problem by a favorite method. The same habit is observed in the present report. Professor Rankine was quoted. A further illustration from his writings will be given: "A cycloidal arc is such a curve; and it has geometrical properties which causes the calculation of its moment of inertia to be a very simple process."[35]

In conclusion, the thanks of the Engineering Fraternity are due to those who framed the Fifth Progress Report of this Committee.

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[33] "A Review of Current Theories", *Engineering News-Record*, Vol. 106, p. 932, June 4, 1932.

[34] *Proceedings*, Am. Soc. C. E., May, 1933, p. 947.

[35] *The Engineer* (London), November 13, 1868, p. 355.

L. E. GRINTER,[36] ASSOC. M. AM. SOC. C. E. (by letter).[36a]—In this enlightening report the Sub-Committee discusses the wind forces on rounded and sloping roofs. The complication involved becomes evident when one observes the tangled mass of curves in Fig. 1. Perhaps greater consistency in results would be evident if some other method of plotting were utilized. The use of the fractional part of a central angle, θ, seems a rather artificial designation, particularly for a sloping roof. There would seem to be quite as much reason to choose the fractional part of the rise or the fractional part of the span. However, brief studies made by the writer did not indicate that either of these designations would be particularly more satisfactory. No attempt was made to study other possibilities, such as the use of an angle with its vertex at one eave, or an angle with its vertex at the ridge.

Pressures on a Rounded Roof.—The Sub-Committee's recommendations in regard to a specified wind pressure for the three segments of the arc of a building roof seem rather complex at first consideration, but actually they are comparatively simple. Some confusion will probably occur in the interpretation of Statement (3) which leads to Equation (6). This statement could be improved since a failure to appreciate the significance of the sentence construction would lead to an erroneous interpretation. Several very intelligent students failed entirely to grasp its significance.

Example of the Goodyear Dock.—The recommendations of the Sub-Committee with reference to wind-pressure distribution on a rounded roof that

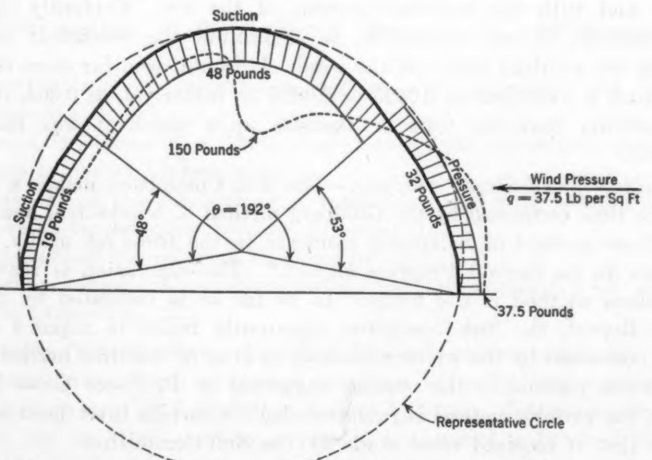

FIG. 6.—WIND PRESSURE DIAGRAM FOR THE GOODYEAR ZEPPLIN DOCK, AT AKRON, OHIO.

extends to the ground level were applied to the Goodyear Zeppelin Dock, at Akron, Ohio. The results obtained are shown in Fig. 6 in which the actual distribution of wind pressure as found by a model test is indicated by a

[36] Prof., Structural Eng., Agri. and Mech. Coll. of Texas, College Station, Tex.
[36a] Received by the Secretary May 29, 1936.

dotted line.[37] It seems evident from this comparison that the recommendations of the Sub-Committee would be reasonably satisfactory for the design of the main structure. However, the observed maximum suction of 150 lb per sq ft, which is three times the specified maximum suction, might influence the selection and means of attaching the roofing material. The maximum specified pressure is $p = 1.4\, r\, q = 1.4 \times \frac{197.5}{325} \times 37.5 = 32$ lb per sq ft. This pressure covers an arc, α, which is found by the formula, $\alpha = k\,\theta = (0.25\, r + 0.125)\, 192 = 53$ degrees. The suction over the windward quarter of the 192° arc is specified as 0.5 q, or 19 lb per sq ft. The suction over the remainder of the roof (approximately the upper half) is specified as $p' = (r + 0.7)\, q = (0.6 + 0.7)\, 37.5 = 48$ lb per sq ft. These values are shown in Fig. 6 where they may be compared with the original test results.

Wind-Pressure Formula.—The recommendation is made that the controlling pressure can be found by Equation (2), $q = 0.002558\, V^2$. The acceptance of this formula without any published justification seems unusual from several points of view. In the first place, engineers are accustomed to the use of a higher coefficient, such as 0.0032, or even 0.004, which naturally gives a much higher pressure for a given wind velocity. Whatever the justification may be for the lower coefficient, the writer feels that it is always a mistake to express empirical or test coefficients in terms of four or even three significant figures. This particular coefficient is one that varies with the barometric pressure and with the moisture content of the air. Evidently, it is also highly difficult, if not impossible, to determine the coefficient accurately because of the swirling action of the wind. It would seem far more reasonable to state such a coefficient as 0.0025 or 0.0026 or, better yet, as 0.003, if there is any possibility that the tests or theories upon which it was based were inadequate.

Simplified Wind-Stress Analysis.—The Sub-Committee makes a comparison of the time consumed by the Goldberg method of wind-stress analysis and by the Cross method of balancing moments in the form set up by the Sub-Committee in its Second Progress Report.[38] The conclusion is reached that the Goldberg method is the faster. In so far as is indicated by the Fifth Progress Report, the Sub-Committee apparently failed to make a study of the time consumed by the writer's method, or even of the time needed to carry out the Cross method in the manner suggested by Professor Cross himself.[39] However, the writer's method is recommended for certain investigations, which indicates that it received some study by the Sub-Committee.

In order to determine the relative usefulness of the two methods recommended by the Sub-Committee, the writer obtained the data on three 8-story bents studied by Professor Young. These bents had been analyzed by the Cross and Goldberg methods, and the time needed in each case will be found

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[37] "Erecting the World's Largest Roof", by Wilbur J. Watson, M. Am. Soc. C. E., *Civil Engineering*, November, 1930, p. 75.

[38] *Proceedings*, Am. Soc. C. E., February, 1932, p. 224.

[39] "Continuous Frames of Reinforced Concrete", by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., 1932 Edition, p. 231.

in the report under the heading, "Goldberg Method." These three bents were assigned to F. J. Benson, Jun. Am. Soc. C. E., for analysis. Mr. Benson had never previously investigated a wind bent, although he understood the fundamental methods of balancing moments, and of slope-deflection as applied to single-story bents. Using the writer's simplified method, he analyzed his first wind bent (8 stories) by estimating deflections, balancing moments, and correcting moments by proportion, in approximately 4½ hr. He later analyzed the two other 8-story bents in about the same length of time. Mr. Benson then analyzed one of these bents by the Goldberg method. He obtained satisfactory results, but the time consumed was in excess of 13 hr. A second study by the Goldberg method, in this case of the lower four stories only, of a typical wind bent, consumed fully 6 hr. The rate, therefore, was but little reduced.

Minimum Time Required for Wind-Stress Analysis.—Since the time consumed by Mr. Benson in applying the writer's method, although considerably shorter than any time recorded by the Sub-Committee, still represented the time needed by an inexperienced operator, the writer decided to repeat the analyses himself. Undoubtedly, the following record of total time will seem unbelievably low to any one who does not understand the method, but it can unquestionably be attained with reasonable experience. It was found that the total time was divided among six major operations, each of which consumed between 15 and 30 min for an 8-story symmetrical building, three bays wide:

(1) Determination of the fixed-end moments by estimating relative deflections;

(2) Computation of distribution factors and recording the fixed-end column moments;

(3) Balancing moments a single time at each joint (moments were carried over at once to hasten convergence);

(4) Second and third balancing of moments (the fourth balance is unimportant, but its introduction requires little time);

(5) Addition of the columns of moments; and,

(6) Determination of criterion ratios and the correction of the final moments by proportion.

Based upon this experience, the writer definitely can state that this method of wind-stress analysis can be applied in rather leisurely fashion by allowing 30 min for each step, or 3 hr for an 8-story building. Naturally, the writer would not try to attain useless accuracy, but would reduce all fixed-end moments (by dividing by a constant) so that they would range between 100 and 300. Convergence will then be quite rapid, and still the accuracy attained will be satisfactory. A thoughtless use of 4-place figures throughout the analysis would probably double the time consumed. The method of balancing moments has been found by experiment to lend itself perfectly to the determination of final moments to any degree of accuracy desired.

Conclusion.—Each of the five reports presented by the Sub-Committee has helped greatly to clarify some group of problems inherent in the design of

buildings to resist wind pressure. Particularly, in having called to attention the tests on rounded roofs, the Sub-Committee will undoubtedly have achieved an improvement in design procedure. Evidently, there is no longer any excuse for the engineer to neglect the suction effect in his design of an arched roof.

Dr. Chr. Nokkentved[40] (by letter).[40a]—For several years the writer, with Dr. J. O. V. Irminger, has studied the characteristics of wind pressure on buildings, particularly variations of pressure. Such variations may be due to: (*a*) The relative size of the object sustaining the wind load; (*b*) the relative air-tightness of the object sustaining the wind load; and (*c*) the influence of adjacent objects.

(a).—The Size of the Model.—The experiments have shown that, under certain circumstances, the pressure distribution on the building, and especially on the windward roof surface, varies considerably with the Reynolds number; that is, it varies partly with the size of the model and partly with the wind velocity.[41] The deciding factor in that case is the relative roughness of the ground; that is, the ratio, $\frac{\delta}{h_w}$, in which $\delta =$ the thickness of the boundary layer adjacent to the ground (calculated from the velocity gradient); and $h_w =$ the characteristic height of the building; that is, the height of the wall.

(b).—The Internal Pressure.—In a building which is uniformly air "leaky" throughout an internal suction of about 0.25 q to 0.35 q will always be exerted. In practice, where the "leakage" that determines the internal pressure may be either on the windward or the leeward sides one has to consider the possibility of varying internal pressure, which may range from pressure to suction, depending upon which surface is being investigated.

(c).—Influence of Adjacent Structures.—C. L. Harris[42], Assoc. M. Am. Soc. C. E., and the writer,[43] have found that neighboring structures may have a remarkably great influence on the distribution of the wind pressure on a building. It will never be possible, therefore, to state definite pressures for the different surfaces of a building. Such values must be stated in terms of an upper and a lower limit.

The different surfaces of the building may be identified as: (*A*) The windward wall; (*B*) the windward roof surface; (*C*) the leeward roof surface; and (*D*) the leeward wall. When the upper and lower limits of wind pressure for these surfaces are to be fixed, the aim is not only to enable a designer to deal with each surface independently, but to design the entire structure by means of the available data.

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[40] Prof., Polyteknisk Læreanstalt, Copenhagen, Denmark.

[40a] Received by the Secretary June 5, 1936.

[41] "Variation of the Wind-Pressure Distribution on Sharp-Edged Bodies", Rept. No. 7. Structural Research Laboratory, Royal Technical Coll., Copenhagen, Denmark, 1936.

[42] "Influence of Neighboring Structures on the Wind Pressure on Tall Buildings", U. S. Dept. of Commerce, National Bureau of Standards, *Research Paper, R. P. 637*, January, 1934, p. 103.

[43] *Ingeniørvidenskabelige Skrifter*, A Nr. 42, Chapter VII

The upper limit for pressures on Surfaces (A) and (B), together with such pressures on Surfaces (C) and (D), should be prescribed so that the pressure on the entire structure will be as nearly identical as possible with the greatest total pressure that the various combinations may produce on the entire structure. Furthermore, a lower limit for all surfaces must be prescribed. This limit may be used for a supplementary calculation of the construction of each surface, as, for example, the fastening of the roofing, roof purlins, and trusses. It will be proper, therefore, to adopt the following pressures:

$$p_A = + 1.00\, q \quad \text{.......... (10)}$$

$$p_B \text{ (for } \alpha > 75°) = + 1.00\, q \quad \text{.......... (11)}$$

$$p_B \text{ (for } \alpha < 75°) = + 2\,\alpha - 50 + (70 - \alpha)\,(1 - w) \quad \text{...... (12)}$$

and,

$$p_C = p_D = - 0.20\, q \quad \text{.......... (13)}$$

in which w = ratio of height of side wall to breadth of building. Furthermore, the limiting upper values for the negative pressure are as determined by the formula, $p = - 0.60\, q$, when q is expressed by Equation (1) of the report.

Considering that a 180° change in the direction of the wind will transfer p_A and p_B Surfaces (D) and (C), it will be obvious that all surfaces must be computed for an upper and a lower value of pressure, and that the purpose of Equation (13) is to enable one to compute the correct total pressure.

In Equations (11) and (12) the influence of the height of the building in relation to its breadth has been taken into account. That this is most necessary is evident from the experiments conducted by the writer.

In former specifications, w has been fixed at 1. If this value of w is inserted in Equations (11) and (12), it is evident that the expression for p_B corresponds very closely to the corresponding value in the Dutch and German specifications.

The writer's proposal differs from the Russian[9], and from that of the Sub-Committee especially in the fact that it yields a greater value of p (less the value of suction) for small values of α. He realizes that the pressures stated for a normal building, freely exposed, and with normal air "leakage", are too great and that those proposed by the Sub-Committee are more correct but considering the possibilities of variation, previously mentioned, the security against various contingencies is greater. Since, at the same time, a lower limit exists, there will be no danger of the actual conditions exceeding the two aforementioned limits.

Furthermore, parts of the structure may be exposed to an even greater suction than that stated herein. However, it is unnecessary to complicate

[9] *Transactions*, Central Aero-Hydrodynamical Inst., U. S. S. R., No. 35, 1928. (In Russian.)

the specifications with details concerning this phase of the problem, since one limit will introduce a smaller factor of safety for secondary parts of the structure, which will result in reckoning with a smaller load than the one that actually occurs.

In this report it is stated that the arithmetic sum of the pressure on the windward wall and the suction on the leeward wall for the Empire State Building, in New York City, is 1.60 q. This might tempt designers to increase $p_A + p_D$ beyond the value of 1.20 q, which is proposed in this discussion. However, it must be considered that $p_A + p_D$, for very tall buildings only, increases beyond the value, 1.20 q.

To shed more light on this question some results of experiments conducted with square prisms with different heights, are shown in Table 2, the side being l and the height, h. These values correspond very well with those found

TABLE 2.—INSTANTANEOUS PRESSURES ON TWO SIDES OF A PRISM

Ratio, $w = \frac{h}{l}$	PRESSURES, IN TERMS OF THE FRACTIONAL PART OF THE VELOCITY PRESSURE, q			Source *
	p_A	p_D	$p_A + p_D$	
1............	+0.76 q	−0.29 q	1.05 q	Copenhagen [14]
2.5..........	+0.80 q	−0.50 q	1.30 q	Copenhagen [14]
3............	+0.80 q	−0.70 q	1.50 q	National Bureau of Standards [11]
5............	....	....	1.55 q	Göttingen [13]
	+0.80 q	−1.11 q	1.91 q	Copenhagen [14]
			2.00 q	Göttingen [13]

* *Proceedings*, Am. Soc. C. E., March, 1936, Footnotes 11, 13, and 14, p. 399.

for the Empire State Building. The height of the building must evidently be more than double the length (the horizontal dimension of the building normal to the direction of the wind) before $p_A + p_D$ exceeds 1.20 q.

For ordinary buildings, therefore, this value is sufficient. For buildings higher than three times the length, the writer proposes values of $p_A = 1.00\,q$ and $p_D = -0.60\,q$.

I. WOUTERS,[44] ESQ. (by letter).[44a]—The attention given in the report of the Sub-Committee to the regulations of the Main Committee for Standardization in The Netherlands, is gratifying. In the paragraph following Fig. 4, the statement is made that Fig. 2 appears to justify a greater severity for the negative pressures (the suctions) on closed buildings with gable roofs of small slopes than is prescribed in Standard N-790 of The Netherlands. It might be useful to give an explanation of the values that were adopted by the Committee that framed the ordinance.

The starting point was the choice of suitable averages for the coefficients, giving positive and negative pressures for the external surfaces, with regard to the "static pressure." The term, "static pressure", as used herein, has

[44] Member, Committee on the Technical Basis for Building Ordinances, Main Committee for Standardization in The Netherlands, The Hague, The Netherlands.

[44a] Received by the Secretary June 10, 1936.

been defined by the National Bureau of Standards.[11] For the purpose mentioned, attention was given to the same sources of information as are cited in Footnotes 7 to 19 of the report. The Netherlands experiments[16] were not numerous. As a result of careful consideration, coefficients for external positive and negative pressures were adopted as indicated in Fig. 7(*a*).

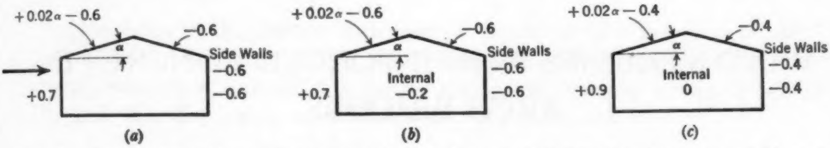

FIG. 7.—BASIS OF WIND PRESCRIPTION FOR GABLE ROOFS, NETHERLANDS STANDARD SPECIFICATION N-790.

The existing internal negative pressure was taken as $-\ 0.2\,q$, thus making the situation as indicated in Fig. 7(*b*). In Fig. 7, the maximum permissible value of α is 65°; for greater values the coefficient is + 0.7 in Fig. 7(*a*) and Fig. 7(*b*), and + 0.9 in Fig. 7(*c*). As only the pressure differences are of interest, it was easier to take the positive and negative pressures with regard to the static pressure $-\ 0.2\ q$, and thus the Committee obtained the coefficients in Standard N-790, III-A, of The Netherlands, which are indicated in Fig. 7(*c*). The allocation of a coefficient, − 0.2, with regard to the "static pressure", for the internal negative pressure is thus a reason for less severity than is indicated in Fig. 2 of the report.

In addition, The Netherlands Committee considered that great suctions act only on a part of the windward roof surface. This was taken into account in Remark 4, of Standard N–790, which, as given in the Second Edition, may be translated as follows:

"Locally, especially where roofs, walls, or both, intersect, suctions much greater than those corresponding to the coefficient, — 0.4, can occur. The designer should be cautious, in general, especially at the places mentioned, to ensure effective fastening of light roof constructions, roofing, and wall materials and sheets."

In the fifth paragraph following Fig. 4 of the report a statement is made that "in The Netherlands regulations, provision is made, in the case of a closed building, for an internal pressure of $-\ 0.4\,q$, * * *." This is not correct, as will appear from what has already been pointed out in this discussion. The internal negative pressure in the case of closed buildings, as adopted in The Netherlands regulations, is $-\ 0.2\ q$ with regard to the "static pressure" and zero with regard to the static pressure $-\ 0.2\ q$, which is also the reference pressure for the other coefficients.

The local maximum positive pressure of $1.2\,q$ was arrived at by adding to the maximum possible external positive pressure, $1.0\,q$, the quantity, $0.2\,q$, arising from the internal negative pressure.

[11] *Research Papers Nos. 301* and *545*, U. S. Bureau of Standards.
[16] *Het Bouwbedrijf*, October 21, 1932.

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DISCUSSIONS

WIND STRESSES IN REINFORCED CONCRETE ARCH BRIDGES

Discussion

BY LOUIS BLUME, ESQ.

LOUIS BLUME,[22] ESQ. (*by* letter).[22a]—Under the heading, "Arch Rib Without Bracing", the author presents an analysis that appears to lead to correct results. However, careful examination indicates that the method of approach is such as to involve superfluous work. To illustrate this, the following treatment of Equations (2), (5), and (6), is presented: Equation (2) can be written,

$$W = \frac{1}{2}\int m^2_u\, dw + \frac{\gamma}{2}\int m^2_v\, dw \quad \ldots\ldots\ldots\ldots (35)$$

Applying the principle of least work:

$$\frac{dW}{dM_w} = \int m_u \frac{dm_u}{dM_w}\, dw + \gamma \int m_v \frac{dm_v}{dM_w}\, dw = 0 \quad \ldots\ldots (36)$$

From Equations (5) and (6), $\frac{dm_u}{dM_w} = +\cos\phi$ and $\frac{dm_v}{dM_w} = -\sin\phi$.

By substitution:

$$\frac{dW}{dM_w} = \int M_w \cos^2\phi\, dw - \int \left[\cos\phi\, dw \int_c^N w\, t\, u\, ds\right]$$

$$+ \gamma \int M_w \sin^2\phi - \gamma \int \left[\sin\phi\, dw \int_c^N w\, t\, v\, ds\right] = 0 \quad \ldots\ldots (37)$$

NOTE.—The paper by A. A. Eremin Assoc. M. Am. Soc. C. E., was published in December, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1936, by Leon Blog, Assoc. M. Soc. C. E.; and August, 1936, by Messrs. Fang-Yin Tsai, and Paul Andersen.

[22] Structural Engr., City Engr.'s Office, Los Angeles, Calif.

[22a] Received by the Secretary August 10, 1936.

Solving for M_w:

$$M_w = \frac{\displaystyle\int \left[\cos\phi \int_c^N w\ t\ u\ ds + \gamma \sin\phi \int_c^N w\ t\ v\ ds\right] dw}{\displaystyle\int [\cos^2\phi + \gamma \sin^2\phi]\ dw} \quad ..(38)$$

This value of M_w agrees with that of Equation (13) but is obtained by a less tortuous procedure. With the value of M_w found, the bending and torsional moments at any point in the rib may be obtained by means of Equations (5) and (6), dispensing with Equations (16) and (19).